Chapter 1

Introduction

1.1 Dusty Plasma

A dynamic (or dusty) plasma is made up of gaseous molecules, electrons, ions, and massive charged dust crystals that range in size from nanometers to microns.A macroscopically neutral gas with numerous interacting charged particles (such as electrons and ions) and neutrals is referred to as plasma. Most likely, 99 percent in our universe exists as a plasma, with dust as one of its ubiquitous constituents. As a result, dust particles typically coexist with a plasma.These particles could have a micron-sized size. They are charged either positively or negatively based on the plasma surroundings around them; they are not neutral.

A "dusty plasma" is created when such charged dust or macroparticles, electrons, ions, and neutrals are mixed together[1] These dust grains, however, can only be a few centimeters in size while in a vacuum. They have a mass of around 106-108 and are easily visible to the naked eye or an alpha camera at the kinetic stage in real time.Although particles are typically powerful, they can also be airy, such as ice crystals or even liquid droplets[2] As a particle is injected into or generated in such a plasma by, say, condensation, the electron or ion flux or by photo, thermo, or secondary electron emission or radioactivity will acquire a positive or negative charge.

1.2 Characteristics

The discovery of plasma crystals in the lab in the middle of the 1990s set off a massive surge in interest in complicated plasmas. Research on the physics of complex plasmas is now expanding quickly and encompasses a number of basic topics, including solid states, hydrodynamics, phase transition kinetics, nonlinear physics, and as well as astrophysical, engineering, and commercial applications. The field is seeing an exponential increase in the number of scholarly publications and the involvement of more and more research organizations worldwide.

1.2.1 Difference between Dusty plasma and Ordinary Plasma

There are three main differences between dusty plasma and ordinary plasmas

1. Formation of Plasmaqq instabilities
2. New dust modes
3. Charge distrubution and density flactuation

1.2.2 Debye Radius

Any individual charged particle in a plasma is shielded by the presence of the surrounding charged particles. This shielding, which takes place at a certain distance, suggests that the surrounding particles either totally or partially reduce the Coulomb interaction between two charged particles. The effective interaction distance between two charged particles in the plasma is represented by this critical distance, also known as the Debye radius. Beyond this range, interactions are considered insignificant. When dust particles are added to the plasma as extra charged particles, a new Debye length unique to the dusty plasma is created.[3]

1.2.3 Modes of Dusty Plasma

In contrast to regular plasmas, dusty plasmas, which are distinguished by the inclusion of dust grains in addition to electrons and ions, display special wave characteristics. The interaction between the dust particles and the plasma waves results in the formation of dust Bernstein-Greene-Kruskal modes, dust-acoustic modes, and dust-lattice modes. Furthermore, dust can alter current plasma wave spectra, changing upper hybrid wave and Langmuir wave frequencies.

1.2.4 Dust Acoustic Waves (DAW)

The system's behavior alters significantly as a result of the charged dust grains, including the emergence of new modes. One such novel mode is the dust acoustic wave (DAW) [4]. Theoretically, dust acoustic waves were first described by Rao et al.'s unmagnetized dusty plasma . Conversely, Shukla and Silin demonstrated the presence of dust ion acoustic waves at higher frequencies .Sound waves known as dust acoustic waves (DAWs) travel through a plasma with charged dust grains acting as inertia and ion and electron pressure acting as a restoring force. In comparison to the thermal velocities of ions and electrons, these waves have a comparatively low phase velocity.[5]

1.2.5 Dust Ion Acoustic Waves (DIAW)

When dust particles are present in dusty plasmas, low-frequency, electrostatic waves known as dust-ion acoustic (DIA) waves are produced. The collective behavior of ions and dust particles is the main cause of these waves' low frequency and propagation. They can be found in a variety of settings, such as comets, planetary rings, and lab plasma equipment.

1.2.6 Momentum exchange in complex plasmas

In complex plasmas, the interchange of momentum between species is quite significant. For instance, the system, especially the grains and ions, is "cooled down" by the momentum transfer in collisions with the neutral gas, which introduces some damping. The forces known as the electron and ion drag forces, which are related to the momentum transfer from electrons and ions to the charged grains, frequently dictate the grain component's static and dynamical characteristics and have an impact on wave events, among other things.[6]

1.3 Plasma Crystals

In a complex (or "dusty") plasma, crystals of plasma can form under specific circumstances. There, a regular macroscopic crystal lattice is formed by the arrangement of electrically charged dust particles. This makes it possible to examine condensed matter's characteristics at the most basic level—kinetics. This implies that fundamental processes, like melting, may be tracked by

looking at how individual particles move. Theoretical and experimental interest in this field of study has skyrocketed since the discovery of plasma crystals in 1994.[7]

An important factor in plasma crystal structure is gravity. In laboratory studies conducted on the ground, primarily two-dimensional crystals can be seen. The sedimentation of the micron-sized particles that accumulate the crystal is the cause of this.

Two prerequisites must be satisfied for plasma crystals to form in a colloidal plasma:

1. The lattice parameter (the ratio of particle separation to Debye length) must be less than unity

2. The Coulomb coupling parameter (the ratio of the Coulomb energy between neighboring particles to their kinetic energy) must surpass a specific threshold. In RF discharge plasmas, these conditions are easily created, and plasma crystallization then happens on its own. Because of their special characteristics, plasma crystals are fascinating systems to research.

3. They may offer several insights into fundamental plasma physical processes and transport effects since they are an as-yet-undiscovered kind of condensed plasma.

4. They may offer fresh insights into solid state physics by serving as model systems for the in-depth study of phase transitions, lattice defects, annealing, doping, etc.

5. They could be helpful as test systems for examining nonlinear effects in "nanocrystals," or crystals with fewer than roughly 100 lattice planes.[8]

1.4 Dusty Plasma in Space

Dusty plasma is everywhere throughout space. Dust grains live and interact with plasma in a wide variety of space environments, such as our solar system, interstellar clouds, circumstantial clouds, etc. The following systems comets, Earth's atmosphere, interplanetary space, planetary rings, etc. where dusty plasma occurs are now briefly detailed. These systems are similar to our solar system.

1.4.1 Comets

They like "dirty snowballs" made of ashes, ice, and stones. In actuality, comets are generated from the solar system's leftover material. They travel to the furthest reaches of the Solar System and in various orbits around the Sun. After years of darkness, only few comets resemble the sun. By reflecting the Sun's light when comets pass by, one may watch comets in the sky.

1.4.2 Planetary Rings

Some of the major planets in our solar system, including Jupiter, Saturn, Uranus, and Neptune, have rings composed of dust particles that are micron and submicron in size. A brief introduction to these planetary rings is given below.

1.4.3 Jupiter Ring

The dust ring system of Jupiter is extremely well-organized and extends across the equatorial region of the planet. It consists of the Thebe (Jupiter XIV) extension, two gossamer rings, and the primary ring. Numerous space missions, including Voyager 1, Voyager 2, Galileo, Cassini, and New Horizons, investigated this dust ring system. Additionally, it was investigated by the Keck observatory on Earth and the Hubble telescope in space [9]. Our knowledge of the dynamical behavior, erosion and impact risks, and contamination of satellite and ring surfaces is improved by studying the dust populations inside Jupiter. In order to reconnoiter the dust environment by evaluating the data obtained from spacecraft, several space missions were launched .

1.4.4 Saturn's Rings

Galileo Galilei made the initial discovery of the Rings of Saturn in 1610. He write below names the seven primary rings that make up this globe. These rings are made up of dust and ice-covered rocks that range in size from a few inches to several meters. Each ring has different dust and plasma characteristics.[10]

The pictures showed what appeared to be ghostly "spokes" moving on the rings, resembling the spokes of a rotating bicycle wheel. Following some conjecture, astronomers concluded that the Saturnian spokes were probably just little dust particles circling the rings as a result of electric and magnetic forces. Plasma is a type of electrically charged gas that produces these force.

1.4.5 Interplanetary Space

It is established that dust particle clouds exist in interplanetary space based on optical data known as zodical radiation. We refer to the area between planets as "Interplanetary Dust" because of the scattered dust particles, even though it is not always empty. Zodiacal light occurs in the inner solar system in addition to the presence of the Asteroids belt. Another uncommon source of interplanetary dust is dust created by collective collisions in the asteroid belt.

By recording the light that dust particles re-emit after being lit by stars, astronomers are able to identify these dusty plasmas in space. It calls for incredibly complex equipment, like the Hubble Space Telescope. The size and chemical makeup of the dust can be deduced from these data.[11]

1.5 Dusty Plasma in Lab

We can only learn a limited amount about natural dusty plasma from a great distance. We can learn more about the properties of dusty plasma by conducting controlled studies in lab settings. For instance, we may directly regulate the dust particle arrangement by adjusting the degree of ionization. The exact arrangement of dust particles in space, on the other hand, cannot currently be inferred from telescopic views.

A laboratory must be able to create, contain, and manipulate an ionized gas that is, one that contains electrically charged particles in order to conduct an experimental investigation of dusty plasma. This is made feasible here at Baylor's CASPER, where we study the behavior of dusty plasma using cutting-edge equipment. For instance, we investigate the interaction of a plasma with lunar dust using a device known as an inductively driven plasma generator, which

is essentially a wind tunnel filled with electrically charged gases.[12]

What occurs in the plasma when extremely fast dust particles strike certain targets is another area of interest. Gaining greater knowledge about these collisions may aid in the development of improved spacecraft protection against micrometeorite impacts.

1.5.1 Microgravity Flights:

Scientists can examine plasma in lower gravity settings via microgravity flights, sometimes referred to as parabolic flights, which are frequently hard or impossible to duplicate on Earth. These trips, which offer brief periods of weightlessness, are accomplished by an airplane flying along a parabolic path. This makes it possible to study complex plasmas, dusty plasmas, and other phenomena where gravity might obfuscate or disrupt the processes under investigation.

1.6 Non Linearity in dusty plasma

One of the most crucial concepts in contemporary physics is a nonlinearity. In addition to offering a range of behaviors intrinsic to turbulence, coherent wave structures, and instabilities, a plasma is rich in nonlinearities.

When perturbations in plasmas are sufficiently large to degrade linear approximations, nonlinear processes take place. These effects result in the creation of harmonics, coupling of various modes, and complex behaviors in plasma parameters. For many applications, it is essential to comprehend nonlinear plasma dynamics. Ponderomotive force, Debye length, Landau damping, and plasma frequency are important ideas. In addition to being essential in wave-particle interactions, instabilities, turbulence, and other real-world plasma phenomena, nonlinear processes can produce coherent structures like solitons and shocks.[13]

1.6.1 Soliton Waves

The soliton wave, which maintains its shape and speed throughout the operation, is a nonlinear wave. A soliton is a localized wave that propagates without altering its velocity or structure.Dispersion and non-linearity are brought into balance as a result.One of Soliton's primary features is that

1) It maintains form.

2)It is localized in the region.

3)When two solitons come into contact with one another, the form and velocity stay the same.

Solitons can be defined as following Kdv partial differential equation.

■■t+ A■■2■x2+ B■■3■■3

where A and B show non- linearity and dispersive concept.

1.6.2 Shock Waves

A propagating disturbance with a very quick increase in temperature, density, and pressure is typically referred to as a shock wave. It has been thoroughly investigated in numerous scientific domains, including simulations of hard-sphere gases using molecular dynamics.The collective action of ions and dust particles is the main cause of these waves' low frequency and propagation. They can be found in comets, planetary rings, and lab plasma devices, among other places.

1.6.3 Methodology to solve nonlinear equations

Under different physical conditions, nonlinear plasma waves obey the governing fluid equations of plasma. Plasma is regarded as one of the scientific frontiers because of the existence of nonlinear plasma inhomogeneity. Many studies have been conducted to better understand soliton propagation, soliton reflection, and other phenomena using the fluid approach in plasma physics. Researchers have been employing the reductive perturbation technique (RPT), one of the primary methods for determining and resolving nonlinear waves in plasma physics, for many decades. Nonlinear waves with tiny amplitudes benefit greatly from this RPT. This method creates a family of nonlinear equations, including the Korteweg-de Vries (KdV) equation, the nonlinear Schrödinger equation (NLSE), and the Kadomtsev Petviashvili (KP) equation, by transforming the space and time coordinates into new extended coordinates.[14]

For DAW waves the governing equations are:

Continuity equation:

Momentum Equation

Electron distribution equation

Ion distribution equation

Poison's equation

1.7 Dust grain charging

In plasma physics, the charging and shielding of an object submerged in plasma is a well-known classic concern. Technically speaking, there is no logical difference between a probe, a dust grain, a spacecraft, and an object in a plasma; however, dusty plasmas generally imply small grain radii with respect to the plasma linearized Debye length. One of the most crucial aspects of complicated plasmas is the particle charge. It establishes how the particle interacts with electromagnetic fields, plasma electrons and ions, other particles, etc. Therefore, a model for the particle charge must be the first step in any study of complicated plasmas.

Electrons may first hit the dust grains' surface when they are submerged in plasma. Any one of them will attach itself to the grain surface, giving it some potential that accelerates the reacted electrons. The temperature of an electron is the main characteristic that defines this procedure[15].

During the charging phase, the grain potential tends to rise until it reaches a point where the Coulomb barrier prevents electrons from penetrating the grain's surface. This happens when the thermal energy of the electrons equals the potential energy, which means that they will be reflected out of the barrier at any distance from the dust grain. However, the pattern is quite clear for ions. They are still attracted to the crop's surface. The potential energy will be reduced as a result of any of them freely falling to the surface and being absorbed, neutralizing some of the grain's negative charge.

During this process, the electrons will once more enter the decreased barrier, strike the surface, and progressively regain their previous potential. To reach equilibrium, at least two processes must be present. Particles far from the grain in the field are constantly moving (Alpert et al. [1964]; Thomas et al. [1994])[16]

One hundred thousand of them will be dominated by dust particles that are a billion times heavier than ions. To do this, one typically needs to determine the floating potential, which is the potential at which the ion and electron currents to the grain specifically cancel one other

out. There is no generally acknowledged explanation for this. There is no straightforward correlation between the dust grain potential and the dust grain charges described above in a plasma, as there is for a charged sphere in a vacuum. Lighter electrons may initially move to the dust grain surface, and the influence of electrons defining the charging phase suggests that the floating potential is found to be KBTe/e.This wakefield is a dynamically screened area of space charge that affects the plasma's general structure as well as the dynamics of other dust particles. The production of plasma crystals and the charging of downstream grains can both be significantly impacted by the wakefield.

1.8 Thesis Layout

In This we dicuss Dusty Plasma some specific and unique characteristics of dusty plasma different modes that are present in dusty plsama . Then we discuss plasma that is present in space with the help of different diagrams and the labortory plasma and how Dusty Plasma experiments carry out.After that we discuss different non linear structures form in dusty plasma.

Chaper 2 deal with the literature survey by different theories to study dusty plasma , reductive purturbation technique and numerical Analysis of Kdv equation.

In chapter 3 we discuss shock waves with viscosity in non maxwallian dusty plasma by reviewing and solving the paper by Dr zahida Ehsan

In chapter 4 we extended the model to study dust size and charge fluctuation in labortory and space plasmas by solving equations.

Chapter 2

Litrature Survey

2.1 Radial Motion (ABR) Theory

In plasma physics, the radial motion of ions in the direction of a probe is described by the Allen-Boyd-Reynolds (ABR) theory. It is suitable to circumstances involving cold ions (where ion temperature is insignificant in comparison to electron temperature) since it is a simplified theory that assumes ions flow directly towards the probe with no additional motion.

It is possible to conclude that the electrons in a repulsive potential obey Boltzmann's law. Therefore, determining the ion density, ni (r), is the issue. In this instance, we examine an ion that travels from the main plasma at velocity vi to the edge of its shielding cloud (r = 1) at velocity vi, arriving at a grain with surface potential at velocity vi, and entering a grain with a radius centred at r = 0. The electrons are satisfied by the Boltzmann law.[17]

Energy Conservation gives

$$\frac{m_i v_i^2}{2} = \frac{m v_i'^2}{2} - e|\varphi_f|, \tag{2.1}$$

where vi indicates the dust particle's velocity in relation to the electrons and ions, and me(i) is the electron's (ion's) mass. The ions are thought to travel radially because they lack angular momentum. The balance of the electron and ion auxes gathered by the particle

$$I_i(R_d) + I_e(R_d) = 0, \tag{2.2}$$

and electron current given [18]

$I_e = -\pi R_d^2 n_{oe} e \left(\frac{8K_B T_e}{\pi m_e}\right)^{1/2} \exp\left(\frac{e\varphi_f}{K_B T_e}\right),$

The floating condition and currents will be

$$\frac{J}{A^2} = \left(\frac{m_i}{4\pi m_e}\right)^{1/2} \exp -|\Phi|, \tag{2.3}$$

Large grain capacity goes to the planar case as expected. There are numerous problems with the ABR idea. First off, angular momentum shouldn't be disregarded for a dust grain with a low potential, even though the model might be quite correct for a heavily distorted spherical probe.

Second, introducing something deep into the plasma is a time-consuming process, and the capacity over a specific radius rather than the current is what you want to know. A shooting code must be used to experiment with different J values if potential for a specified radius is required. Third, resolving the boundary conditions and making sure they are sufficiently distant from the grain adds an extra layer of complexity to the solution that is absent in the flat case.[19]

More complex models of dust charge are provided by other theories, such as the orbital motion limited (OML) theory, which takes into account intricate ion dynamics and absorption mechanisms.

2.2 Orbit Motion (OM) Theory

The dust charging theory, which gives the dust potential and charge resulting from the dust interaction with a plasma, is the fundamental theory for dusty plasmas. The orbital motion limited (OML) theory is the most popular dust charging theory for negatively charged dust particles. It accurately predicts dust potential and heat collection for a range of applications, but it was previously discovered that it is unable to assess dust charge and plasma response under any circumstances. Here, we present an updated OML formulation that can forecast the dust charge and, consequently, the plasma reaction.

The theory of Orbit Motion Limited (OML), in which ions have limited orbits, is the most

basic definition of the OM theory (Suits and Way 1961). This technique determines the cross sections for electrons using just the laws of conservation of energy and angular momentum.

and the dust particle's accumulation of ions. The state of application of the OML theory is Rd << ˜D << li(e), where li(e) is the mean free path of the ions (electrons), indicating that the dust particle is isolated in such a way that the motion of electrons and ions in its vicinity is not influenced by dust particles. In contrast to the ABR theory, the OML theory specifies the obtained limiting ion orbit and addresses the case of hot ions [20].The idea has several shortcomings and is extremely simplistic in contrast to the ABR theory. (Ehsan and others, 2011). At the edge of its shielding cloud (r = 1), we observe an ion entering a grain from the bulk plasma at velocity vi and using surface potential f to reach the grain at velocity vi. Similar to the ABR theory's earlier example, the electrons once more follow the Boltzmann law. The formula for Angular Momentum Conservation is

For a Maxwellian plasma, the ions current can be written as

$$I_i = \pi R_d^2 Z_i n_{oi} e \left(\frac{8K_B T_i}{\pi m_i}\right)^{1/2} \left[1 + \frac{e|\varphi_f|}{K_B T_i}\right]. \tag{2.4}$$

Using the floating condition provided by Eq. (1.2), we get

$$\left(\frac{T_e}{m_e}\right)^{1/2} \exp\left(-\frac{e|\varphi_f|}{K_B T_e}\right) = \left(\frac{T_i}{m_i}\right)^{1/2} \left(1 + \frac{e|\varphi_f|}{K_B T_i}\right). \tag{2.5}$$

According to Allen et al. [2000], the OML hypothesis for Maxwellian plasmas is incorrect, at least in the case of Ti Te. Although extremely uncommon, this may happen in a fusion system. For dust plasma considerations, it was later discovered to be true [21]. Dusty plasma scientists have applied this theory extensively in the field of dusty plasmas.

2.3 Adiabatic model

Throughout the charging procedure, the specimen's temperature fluctuates . The temperature will gradually change in space and time even if the grain is at rest and the electrons and ions only have thermal energy because of sporadic particle collisions and collisions with the grain surface. Another process that might cause a temperature change is the thermal adduction on

the grain surface brought on by the particles' absorption and re-emission. This suggests that the system is processing adiabatically rather than isothermally [22].

Therefore, the density may be used to represent the pressures for the electrons (Pe) and ions (Pi)

$$P_e = n_{0e}T_{oe}\left(\frac{n_e}{n_{0e}}\right)^{5/3}; P_i = n_{0i}T_{0i}\left(\frac{n_i}{n_{0i}}\right)^{5/3}, \tag{2.6}$$

and where $n_{o\alpha}$and $T_{o\alpha}$are the mean density and temperature of the species(e, i), we obtain for the densities of electrons and ions

$$\frac{n_e}{n_{0e}} = \left(1 - \frac{2}{5}\frac{e|\varphi_{_D}|}{T_{0e}}\right)^{3/2}; \frac{n_i}{n_{0i}} = \left(1 + \frac{2}{5}Z_i\frac{e|\varphi_{_D}|}{T_{0i}}\right)^{3/2}. \tag{2.7}$$

The Poisson equation for the electrostatic potential field can be written as $\nabla^2|\varphi_{_D}| = 4\pi e(Z_i n_i - n_e)$.

The grain surface is easily battered by electrons since they are light particles with thermal velocities higher than ions. Thus, the charging process continues until the potential field of the surface is in the same order as the the thermal kinetic energy of electrons. Put differently, the charge on the surface is determined by the temperature of the electrons (Tsintsadze et al. [2006]).

A useful relation between Z_d and T_e

Dust grainbn barrier does not break by electron due to its lower potential energy. The expression (1.12) of the electron density, from which the full potential field at the surface follows, explains this situation well (electrons are negligible near the surface so that $(n_e/n_{0e}) \to 0$). Thus, $|\varphi_{_D}|_{Max} = 2.5\ T_{oe}/e$,In the other hand, the potential field on the top of the dust grain is, $|\varphi_{_D}|_{Max} = Z_d e/R_d$.Combining these relations, for a given radius of the dust grain, we get a very critical relationship between the charge number and the temperature:

$$Z_d = 2.5\frac{T_e(ev)}{e^2}R_d. \tag{2.8}$$

This determines the dust grain's surface charge precisely and is true as long as the mean distance between dust grains is greater than the Debye length.Using separate T_e and $R_D$values,

the magnitude of the charge$Z_d$of Eq.(1.14) was found to be in strong accordance with the values quoted in the Mendis Table[23].

2.4 Reductive Purturbation technique

One extremely effective technique for creating reduced models that explain the propagation and interaction of nonlinear waves is the reductive perturbation method. We outline the foundations of the approach in abstract frames selected for clarity's sake: long-wave envelope equations

three-wave resonant interaction approximation. We provide an understanding of the perturbative schemes' mathematical characteristics. Next are several applications that have their own physical importance and either show the usual scenario or add new aspects of perturbative expansions. The applications are to either wave propagation in ferromagnetic substances, often known as the electromagnetic or polariton range, or nonlinear optics, particularly ultrafast.

Mathematical ideas that make sense only for linear problems are closely related to the physical concepts involved in the study of linear or linearized phenomena: regularity,

mode, polarization.Using a perturbation method, the first order of which is the linear approximation: the slowly varying envelope approximation, is the most intuitive approach and, technically, the most economical. According to mathematical language, it can therefore be described as "weakly nonlinear"[24].

2.5 Numerical Analysis Of Kdv

First presented in 1895, the Korteweg-de Vries (KdV) equation is a nonlinear PDE that models low amplitude water waves in shallow, narrow channels, such as canals

ut

+ 6uux + uxxx = 0

Since then, it has been used to solve numerous additional physics and engineering issues, such as those involving plasma physics. KdV has been the subject of a great deal of theoretical and numerical research. For the equation, numerous finite difference techniques have been developed, including an explicit one by Zabusky and Kruskal, who found that solitons exist, as

well as an implicit one from Goda

Because KdV has a smooth analytical solution provided by, it is easy to measure the correctness of a particular approach.[25]

$$u(t,x) = \frac{c}{2}\sec h^2[\frac{\sqrt{c}}{2}(x - ct - b)$$

The primary characteristic of KdVE is that the speed of a single wave depends on its magnitude, in contrast to Schrödinger-type equations where the wave speed is independent of the wave amplitude. One unique characteristic of this equation is that its solutions may show a single wave.

solutions called solitons that, following interaction, maintain their initial dimensions, morphology, kortewig, de vries, and velocity. Kortewig and de Vries were the first to introduce KdVE [1].

Numerous physical phenomena have been discovered to be described by this equation, including bubble-liquid mixes, ion acoustic plasma waves, shallow water waves, and wave phenomena in enharmonic crystals.[26]

Chapter 3

Shock waves with viscosity in non-Maxwelllian dusty plasma

Here i review that the research paper by Dr Zahida Ehsan about shock waves with viscosity in non maxwallian plasma .

3.1 Physical assumptions and description of the model

The following basic assumptions are made that will help to formulate the physical problem and to find the results:

1. The considered magnetized dusty plasma is unbounded, homogeneous, and collisionless. Electrons (n_{e}), ions (n_{i}), and negatively charged dust grains (n_{d}), which have a continuous negative charge, are the components of plasma. The requirement for quasineutrality is provided by

$$n_i = n_e + Z_d n_d \tag{1}$$

2. In order to address this issue, they first look at the slow time (DAW) dynamics after the fast time scale event. While the dust species is activated in the latter regime, the dust dynamics are disregarded in the former.

3. With z as the unit vector along the z-axis and B■ as the magnetic field's strength, the ambient magnetic field B=B■z is directed along the z-axis. Since the gyroradius of plasma

species particles is significantly larger than the wavelength ■=2■/k, only dust grains are magnetized, while electrons and ions are not. Additionally, we know that in a weak magnetic field, the electron gyroradius is much smaller than the dust size and the change in dust charge is too small. As a result, electrons move quickly toward the dust grain surface along the direction of the external magnetic field, which may result in a Boltzmann distribution for fast electrons charging a grain.

4. In magnetized plasma for low frequecy when $\omega/k \ll v_{ts}$ the electrons and ions can be treated as Boltzmann distributed.

5. They aim to adopt kappa and Cairns distribution for the lighter species. The normalized number density for kappa and Cairns distributed particles respectively is given by

$$n_{e(i)} = \left[1 \mp \left(\kappa - \frac{3}{2}\right)^{-1} \frac{\phi}{\sigma}\right]^{-\kappa+1/2} \tag{2}$$

and

$$n_{e(i)} = \left[1 \mp \frac{\beta}{\sigma}\phi \pm \frac{\beta}{\sigma^2}\phi^2\right] e^{\pm\phi} \tag{3}$$

where κ is the spectral index measuring the deviation form Maxwellian distribution and $\beta = 4\alpha/(1+3\alpha)$. For the electrons case $\sigma = 1$. For $\kappa \to \infty(\alpha = 0)$ Maxwellian distribution is achieved.

6. Assume that the dust charge is constant because the occurrence time for the nonlinear processes (dust acoustic shock wave) stated is significantly shorter than that needed for a further significant change in dust charge.

7. As mentioned in the introduction, we are thinking about rotating plasmas. It is important to note that we will only (in the case of slow time scale) consider dust particles to be rotating, while we do not care about the rotation of other constituents like electrons and ions. We also assume that the magnetic axis and a rotational axis are not aligned, creating an inclination angle ■ with the magnetic axis that is assumed to be small.[27]

3.2 Fast time scale

Dust particles are regarded as a stable and immobile component in the plasma backdrop because of their enormous bulk in relation to the other constituents in this plasma; their existence is only taken into consideration by the quasineutrality requirement. The hydrodynamic equations for ions and electrons in the presence of a magnetic field are expressed as

$$\frac{\partial n_i}{\partial t} + \boldsymbol{\nabla} \cdot (n_i \mathbf{v}_i) = 0 \tag{4}$$

$$\left(\frac{\partial}{\partial t} + \mathbf{v}_i \cdot \boldsymbol{\nabla}\right) \mathbf{v}_i = -\frac{m'}{Z_d \alpha_d} \boldsymbol{\nabla} \phi + \omega_{ci} (\mathbf{v_i} \times \hat{\mathbf{z}}) + - \frac{\sigma m'}{Z_d \alpha_d} \frac{\boldsymbol{\nabla} n_\mathbf{i}}{n_i} + \eta_i \nabla^2 \mathbf{v_i} + (\eta_i + \mu_i) \boldsymbol{\nabla} (\boldsymbol{\nabla} \cdot \mathbf{v}_i) \tag{5}$$

$$\nabla^2 \phi = \alpha_d (\mu n_{e1} - \delta n_{i1}) \tag{6}$$

Here we define $m' = m_d/m_i$ as the dust to ion mass ratio, $\mu = n_{eo}/n_{do} Z_d$ and $\delta = n_{io}/n_{do} Z_d$ as the ratio of densities whereas $\sigma = T_i/T_e$ as temperature ratio of ions to electrons. For mathematical convenience, we perform the following normalizations. The ion (electron) densities by their respective unperturbed densities, the speeds are normalized by $\omega_{ci} = eB_0/m_i c$ normalized by the $\omega_{pi} = \left(4\pi n_{d0} e^2 Z_d^2/m_d\right)^{1/2}$. The spatial variables in fluid equations are normalized by the $\lambda_d = \left(T_e \alpha_d / 4\pi n_{d0} e^2 Z_d\right)^{1/2}$and the time t by ω_{pd}^{-1}. Moreover, η_i and μ_i are normalized by $\omega_{pd} \lambda_d^2$.

Here η_i and μ_i represent, respectively, is the kinematic viscosity and ion bulk viscosity for ions (also called the second coefficient viscosity). The ion bulk viscosity can only be neglected while discussing incompressible fluids. However, for ion acoustic waves, discussed here, the plasma compressibility is essentially this μ_i term cannot be neglected. The set of equations (6)-(8) in the Cartesian components for 3D nonlinear IA shock waves can be written as

$$\frac{\partial n_i}{\partial t} + \frac{\partial (n_i v_{ix})}{\partial x} + \frac{\partial (n_i v_{iy})}{\partial y} + \frac{\partial (n_i v_{iz})}{\partial z} = 0 \tag{7}$$

$$\begin{aligned} \frac{\partial v_{ix}}{\partial t} + v_{ix} \frac{\partial v_{ix}}{\partial x} + v_{iy} \frac{\partial v_{ix}}{\partial y} + v_{iz} \frac{\partial v_{ix}}{\partial z} = -\frac{m'}{Z_d \alpha_d} \frac{\partial \phi}{\partial x} + \omega_{ci} v_{iy} - \frac{m' \sigma}{Z_d \alpha_d n_i} \frac{\partial n_i}{\partial x} + \eta_i \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) v_{ix} \\ + (\eta_i + \mu_i) \left(\frac{\partial^2 v_{ix}}{\partial x^2} + \frac{\partial^2 v_{iy}}{\partial x \partial y} + \frac{\partial^2 v_{iz}}{\partial x \partial z}\right) \end{aligned} \tag{8}$$

$$\frac{\partial v_{iy}}{\partial t} + v_{ix}\frac{\partial v_{iy}}{\partial x} + v_{iy}\frac{\partial v_{iy}}{\partial y} + v_{iz}\frac{\partial v_{iy}}{\partial z} = -\frac{m'}{Z_d\alpha_d}\frac{\partial \phi}{\partial y} - \omega_{ci}v_{ix} - \frac{m'\sigma}{Z_d\alpha_d n_i}\frac{\partial n_i}{\partial y} + \eta_i\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)v_{iy}$$
$$+ (\eta_i + \mu_i)\left(\frac{\partial^2 v_{ix}}{\partial y\partial x} + \frac{\partial^2 v_{iy}}{\partial y^2} + \frac{\partial^2 v_{iz}}{\partial y\partial z}\right) \tag{9}$$

$$\frac{\partial (v_{iz})}{\partial t} + v_{ix}\frac{\partial (v_{iz})}{\partial x} + v_{iy}\frac{\partial (v_{iz})}{\partial y} + v_{iz}\frac{\partial (v_{iz})}{\partial z} = -\frac{m'}{Z_d\alpha_d}\frac{\partial \phi}{\partial z} - \frac{m'\sigma}{Z_d\alpha_d n_i}\frac{\partial n_i}{\partial z} + \eta_i\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)v_{iz}$$
$$+ (\eta_i + \mu_i)\left(\frac{\partial^2 v_{ix}}{\partial z\partial x} + \frac{\partial^2 v_{iy}}{\partial z\partial y} + \frac{\partial^2 v_{iz}}{\partial z^2}\right) \tag{10}$$

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = \alpha_d\left(\mu + c_1\phi + c_2\phi^2 - \delta n_{i1}\right) \tag{11}$$

$$c_1 = \begin{cases} \mu(1-\beta) \ \text{(Cairns)}, \\ \mu\left(\dfrac{\kappa - 1/2}{\kappa - 3/2}\right) \ \text{(kappa)} \end{cases}$$
$$c_2 = \begin{cases} \mu/2 \ \text{(Cairns)}, \\ \dfrac{\mu(\kappa - 1/2)(\kappa + 1/2)}{2(\kappa - 3/2)^2} \ \text{(kappa)} \end{cases} \tag{12}$$

We employe well-known reductive perturbation method [28] to study shock waves of small (but finite) amplitude also we use stretched coordinates given as:

$$\xi = \epsilon^{1/2}x, \qquad \eta = \epsilon^{1/2}y,$$
$$\zeta = \epsilon^{1/2}(z - \lambda_0 t) \ \text{and} \ \tau = \epsilon^{3/2}t \tag{13}$$

where λ_0 is denoting the wave propagation speed and ϵ $(0 < \epsilon \ll 1)$ is a small (dimensionless) parameter which measures the strength of nonlinearity the weakness. Further, the dependent

variables are expanded in power series of ϵ, i.e.

$$
\begin{aligned}
n_i &= 1 + \varepsilon n_1 + \epsilon^2 n_2 + ...,\\
v_{ix} &= \epsilon^{3/2} u_1 + \epsilon^2 u_2 + ...,\\
v_{iy} &= \epsilon^{3/2} v_1 + \epsilon^2 v_2 + ...,\\
v_{iz} &= v_0 + \epsilon w_1 + \epsilon^2 w_2 + ...,\\
\phi &= \epsilon \phi_1 + \epsilon^2 \phi_2 + ..., \qquad (14)
\end{aligned}
$$

The ion kinematic viscosity, for weakly damped systems, is assumed to be small, and can be expressed as

$$\eta_i = \epsilon^{1/2} \eta_0, \quad \mu_i = \epsilon^{1/2} \mu_0, \tag{15}$$

where η_0 and μ_0 are finite quantities of the order unity. Upon using Eqs. (15)-(17) in Eqs. (9)-(13) and comparing the terms of different powers of ϵ we can evaluate the variation properties of wave, e.g., the lowest order in ϵ ($\sim O(\epsilon)$ and $O(\epsilon^{3/2})$) results in

$$w_1 = n_1 \left(\lambda_o - v_o\right) \tag{16}$$

$$n_1 = \frac{c_1}{\delta} \phi_1 \tag{17}$$

$$\frac{m'}{Z_d \alpha_d} \frac{\partial \phi_1}{\partial \xi} + \frac{\sigma m'}{Z_d \alpha_d} \frac{\partial n_1}{\partial \xi} - \omega_{ci} v_i = 0 \tag{18}$$

$$\frac{m'}{Z_d \alpha_d} \frac{\partial \phi_1}{\partial \eta} + \frac{\sigma m'}{Z_d \alpha_d} \frac{\partial n_1}{\partial \eta} + \omega_{ci} u_i = 0 \tag{19}$$

$$-\left(\lambda_o - v_o\right) \frac{\partial w_1}{\partial \zeta} + \frac{\sigma m'}{Z_d \alpha_d} \frac{\partial n_1}{\partial \zeta} + \frac{m'}{Z_d \alpha_d} \frac{\partial \phi_1}{\partial \zeta} = 0 \tag{20}$$

From above equations, we obtain linear dispersion relation for the DMSW

$$\lambda_0 = v_0 \pm \sqrt{\frac{m'}{Z_d \alpha_d c_1} \left(\sigma c_1 + \delta\right)} \tag{21}$$

Eq. (23) provides the phase speed for IA waves, where, the $+(-)$ sign corresponds to the fast (slow) IA speeds. Moving farther to next order in ϵ ($\sim O(\epsilon^2)$ and $O(\epsilon^{5/2})$) results in 2^{nd} order

perturbed quantities, as following

$$\left(\lambda_0 - v_0\right)\frac{\partial u_1}{\partial \zeta} + \omega_{ci} v_2 = 0 \tag{22}$$

$$\left(\lambda_0 - v_0\right)\frac{\partial v_1}{\partial \zeta} = \omega_{ci} u_2 \tag{23}$$

$$\frac{\partial^2 \phi_1}{\partial \xi^2} + \frac{\partial^2 \phi_1}{\partial \eta^2} + \frac{\partial^2 \phi_1}{\partial \zeta^2} = \alpha_d \left(c_1 \phi_2 + c_2 \phi_1^2 - \delta n_2\right) \tag{24}$$

$$-\left(\lambda_0 - v_0\right)\frac{\partial n_2}{\partial \zeta} + \frac{\partial u_2}{\partial \xi} + \frac{\partial v_2}{\partial \eta} + \frac{\partial w_2}{\partial \zeta} = -\frac{\partial n_1}{\partial \tau} - \frac{\partial}{\partial \zeta}\left(n_1 w_1\right) \tag{25}$$

From Eqs. , we obtain the ZKB equation:

$$\frac{\partial \phi_1}{\partial \tau} + A\phi_1 \frac{\partial \phi_1}{\partial \zeta} + B\frac{\partial^3 \phi_1}{\partial \zeta^3} + C\frac{\partial}{\partial \zeta}\left(\frac{\partial^2 \phi_1}{\partial \xi^2} + \frac{\partial^2 \phi_1}{\partial \eta^2}\right) - D\left(\frac{\partial^2 \phi_1}{\partial \xi^2} + \frac{\partial^2 \phi_1}{\partial \eta^2} + \frac{\partial^2 \phi_1}{\partial \zeta^2}\right) - E\frac{\partial^2 \phi_1}{\partial \zeta^2} = 0 \tag{26}$$

In deriving above equation the dynamics of dust particles is gnored, it's contribution comes only from the quasineutrality condition. To find the solution of ZKB equation (29), let us introduce the parameter χ as

$$\chi = l_x \xi + l_y \eta + l_z \zeta - U_0 \tau \tag{27}$$

here $l_\alpha (\alpha = x, y, z)$ denotes the direction cosines and U_0 represents the nonlinear wave speed. Using Eq. (34) into (29) yields the following ordinary differential equation (ODE) as

$$-U_0 \frac{d\phi_1}{d\chi} + A l_z \phi_1 \frac{d\phi_1}{d\chi} + H l_z \frac{d^3 \phi_1}{d\chi^3} - G\frac{d^2 \phi_1}{d\chi^2} = 0, \tag{28}$$

where $H = l_z^2 B + \left(l_x^2 + l_y^2\right) C$ and $G = E l_z^2 + D$. The shock like solution of Eq. (35) can be found the hyperbolic tangent method . Thus, employing the condition that ϕ_1 is bounded at $\chi = \pm\infty$, we obtain

$$\phi_1(\chi) = \frac{3}{25}\,\frac{G^2}{H A l_z^2}\left[2 - 2\,\tanh\left(\frac{G}{10 H l_z}\chi\right) + \mathrm{sech}^2\left(\frac{G}{10 H l_z}\chi\right)\right] \tag{29}$$

It is an example of a ZKB equation shock wave solution. Thus, the ion kinematic viscosity term causes the shock-like solution to arise.. Here, $10 H l_z / G$ and $(9/25)\left(G^2 / H A l_z^2\right)$ represent the width and amplitude of the shock structure, respectively, which depends upon the dispersive

coefficients B and C, dissipative coefficients D and E, and direction cosines. The amplitude of the wave also depends on the nonlinear coefficient A. These coefficient, which depend on various plasma parameter, determines the shape of the shock potential profile.

3.3 Dust acoustic (DA) wave at slow time scale

Now we derive a linear dispersion relation for the ultralow frequency

Now we take into account the quasi-neutrality condition $\delta n_i \sim Z_d \delta n_d$, since the time with which velocity and density of electrons changes is much shorter than that of ions and dust, i.e.,

$$v_e \left(\frac{\partial v_e}{\partial t}\right)^{-1}, n_e \left(\frac{\partial n_e}{\partial t}\right)^{-1} << t_i \sim \frac{1}{\omega_{pi}}, t_d \sim \frac{1}{\omega_{pd}} \tag{30}$$

where ω_{pi} and ω_{pd} are the Langmuir frequencies for ions and dust, respectively, whereas the neutrality condition ($n_{io} = Z_d n_{do} + n_{eo}$) holds. In this case, the dynamic effects of the dust grains are included, because we are interested in the dust's time and space scales. For that, we know that $\delta n_e << \delta n_i, \delta n_d$,

$$\frac{\partial n_d}{\partial t} + \boldsymbol{\nabla} \cdot (n_d \mathbf{v}_d) = 0 \qquad 31$$

$$\left(\frac{\partial}{\partial t} + \mathbf{v}_d \cdot \boldsymbol{\nabla}\right) \mathbf{v}_d = \frac{1}{\alpha_d} \boldsymbol{\nabla}\phi - \omega_{cd} \left(\mathbf{v_d} \times \hat{\mathbf{z}}\right) - \frac{\sigma_d}{\alpha_d} \frac{\boldsymbol{\nabla} \mathbf{n_d}}{n_d} + 2\Omega_o \left(\mathbf{v_d} \times \hat{\mathbf{z}}\right) + \eta_d \nabla^2 \mathbf{v_d} + (\eta_d + \mu_d) \boldsymbol{\nabla} \left(\boldsymbol{\nabla} \cdot \mathbf{v}_d\right) \tag{32}$$

$$\nabla^2 \phi = \alpha_d \left(\mu n_e - \delta n_i + n_d\right) \tag{33}$$

where $\sigma_d = T_d/T_e Z_d$ is ion to electron temperature ratio. Ω_o is the rotational frequency of the dust. The above equations are normalized as follows: The ion electron number densities n_i and n_e are normalized by their unperturbed densities, respectively, the ion fluid velocity $\mathbf{v}_i$ and the speed of light c are normalized by ion acoustic speed $c_s = (Z_d T_e \alpha_d / m_d)^{1/2}$, the electrostatic potential ϕ is normalized by T_e/e, the ion gyrofrequency $\omega_{cd} = e Z_d B_0 / m_d c$ normalized by the $\omega_{pi} = \left(4\pi n_{d0} e^2 Z_d^2 / m_d\right)^{1/2}$. The space variables are normalized by the Debye length $\lambda_d = \left(T_e \alpha_d / 4\pi n_{d0} e^2 Z_d\right)^{1/2}$and the time t by the inverse of the ion plasma frequency ω_{pd}^{-1}. Also, the ion kinematic viscosity η_i and the second coefficient of viscosity μ_i are normalized by $\omega_{pd} \lambda_d^2$.[29]

The ions and electrons are considered to obey non-Maxwellian distributions namely Kappa and Cairns distributions. The normalized number density for ions and electrons obeying kappa

distribution are given by Eqs. (4) and (5).

The set of governing equations (38) - (40) for three dimensional nonlinear dust acoustic shock waves can be expressed in the Cartesian components form as follows

$$\frac{\partial n_d}{\partial t}+\frac{\partial\left(n_d v_{dx}\right)}{\partial x}+\frac{\partial\left(n_d v_{dy}\right)}{\partial y}+\frac{\partial\left(n_d v_{dz}\right)}{\partial z}=0 \tag{34}$$

$$\begin{aligned}\frac{\partial v_{dx}}{\partial t}+v_{dx}\frac{\partial v_{dx}}{\partial x}+v_{dy}\frac{\partial v_{dx}}{\partial y}+v_{dz}\frac{\partial v_{dx}}{\partial z}=\frac{1}{\alpha_d}\frac{\partial \phi}{\partial x}-\Omega_c v_{dy}-\frac{\sigma_d}{\alpha_d n_d}\frac{\partial n_d}{\partial x}+\eta_d\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)v_{dx}\\+\left(\eta_d+\mu_d\right)\left(\frac{\partial^2 v_{dx}}{\partial x^2}+\frac{\partial^2 v_{dy}}{\partial x\partial y}+\frac{\partial^2 v_{dz}}{\partial x\partial z}\right)\end{aligned} \tag{35}$$

$$\begin{aligned}\frac{\partial\left(v_{dz}\right)}{\partial t}+v_{dx}\frac{\partial\left(v_{dz}\right)}{\partial x}+v_{dy}\frac{\partial\left(v_{dz}\right)}{\partial y}+v_{dz}\frac{\partial\left(v_{dz}\right)}{\partial z}=\frac{1}{\alpha_d}\frac{\partial \phi}{\partial z}-\frac{\sigma_d}{\alpha_d n_d}\frac{\partial n_d}{\partial z}+\eta_d\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)v_{dz}\\+\left(\eta_d+\mu_d\right)\left(\frac{\partial^2 v_{dx}}{\partial z\partial x}+\frac{\partial^2 v_{dy}}{\partial z\partial y}+\frac{\partial^2 v_{dz}}{\partial z^2}\right)\end{aligned} \tag{36}$$

$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}+\frac{\partial^2\phi}{\partial z^2}=\alpha_d\left(+c_{d1}\phi+c_{d2}\phi^2-1+n_d\right) \tag{37}$$

where we define $\Omega_c=\omega_{cd}-2\Omega_o$ and $\mu-\delta=1$, comes from the charge neutrality condition.

$$\begin{aligned}c_{d1}&=\begin{cases}\left(\mu-\frac{\delta}{\sigma}\right)(1+\beta)\ \text{(Cairns)},\\ \frac{\mu(\kappa-1/2)}{\kappa-3/2}+\frac{\delta(\kappa-1/2)}{(\kappa-3/2)\sigma}\ \text{(kappa)}\end{cases}\\ c_{d2}&=\begin{cases}\frac{\mu}{2}-\frac{\delta(1+4\beta)}{2\sigma^2}\ \text{(Cairns)},\\ \left(\mu-\frac{\delta}{\sigma^2}\right)\frac{\mu(\kappa-1/2)(\kappa+1/2)}{2(\kappa-3/2)^2}\ \text{(kappa)}\end{cases}\end{aligned} \tag{38}$$

To study the ion acoustic shock waves of small, but finite amplitude, we use the standard reductive perturbation method to obtain the ZK-Burgers equation. According to this approach, we use the stretching for the independent space and time variables as

$$\begin{aligned}&\xi=\epsilon^{1/2}x, \qquad \eta=\epsilon^{1/2}y,\\ &\zeta=\epsilon^{1/2}\left(z-\lambda_0 t\right)\ \text{and}\ \tau=\epsilon^{3/2}t\end{aligned} \tag{39}$$

where λ_0 is the normalized wave propagation speed to be determined later and ϵ $(0 < \epsilon \ll 1)$ is a small dimensionless parameter determining the weakness of the dispersion and nonlinearity. Furthermore, the dependent variables n_i, $\mathbf{v}_i$,and ϕ are expanded in power series of ϵ as

$$\begin{aligned} n_d &= 1 + \varepsilon n_{d1} + \epsilon^2 n_{d2} + ..., \\ v_{dx} &= \epsilon^{3/2} u_{d1} + \epsilon^2 u_{d2} + ..., \\ v_{dy} &= \epsilon^{3/2} v_{d1} + \epsilon^2 v_{d2} + ..., \\ v_{dz} &= v_{d0} + \epsilon w_{d1} + \epsilon^2 w_{d2} + ..., \\ \phi &= \epsilon \phi_1 + \epsilon^2 \phi_2 + ..., \end{aligned} \tag{40}$$

For weak damping, the ion kinematic viscosity is assumed to be small, so that

$$\begin{aligned} \eta_d &= \epsilon^{1/2} \eta_{d0} \\ \mu_d &= \epsilon^{1/2} \mu_{d0} \end{aligned} \tag{41}$$

where η_{d0} and μ_{d0} are finite quantities of the order of unity. On substituting Eqs. (41) - (45) into Eqs. (47) - (49) and collecting the terms in different powers of ϵ, the lowest order in ϵ ($\sim O(\epsilon)$ and $O(\epsilon^{3/2})$) gives the following equations:

$$w_{d1} = n_{d1} \left(\lambda_{do} - v_{do} \right) \tag{42}$$

$$n_{d1} = -c_{d1} \phi_1 \tag{43}$$

$$\frac{1}{\alpha_d} \frac{\partial \phi_1}{\partial \xi} - \frac{\sigma_d}{\alpha_d} \frac{\partial n_{d1}}{\partial \xi} - \Omega_c v_{d1} = 0 \tag{44}$$

$$\frac{1}{\alpha_d} \frac{\partial \phi_1}{\partial \eta} - \frac{\sigma_d}{\alpha_d} \frac{\partial n_{d1}}{\partial \eta} + \Omega_c u_{d1} = 0 \tag{45}$$

$$-\left(\lambda_{do} - v_{do} \right) \frac{\partial w_{d1}}{\partial \zeta} - \frac{\sigma_d}{\alpha_d} \frac{\partial n_{d1}}{\partial \zeta} + \frac{1}{\alpha_d} \frac{\partial \phi_1}{\partial \zeta} = 0 \tag{46}$$

From equations (51-54), we obtain

$$\lambda_{d0} = v_{d0} \pm \sqrt{\frac{1 + \sigma_d c_{d1}}{\alpha_d c_{d1}}} \tag{47}$$

Equation (55) represents the phase speed for dust acoustic waves, whereas, the $\pm$ sign corresponds to the fast and slow ion acoustic speeds. The next higher order in ϵ ($\sim O(\epsilon^2)$ and $O(\epsilon^{5/2})$) gives a set of equations in the second order perturbed quantities, given by

$$(\lambda_{d0} - v_{d0}) \frac{\partial u_{d1}}{\partial \zeta} - \Omega_c v_{d2} = 0 \tag{48}$$

$$(\lambda_{d0} - v_{d0}) \frac{\partial v_{d1}}{\partial \zeta} + \Omega_c u_{d2} = 0 \tag{49}$$

$$\frac{\partial^2 \phi_1}{\partial \xi^2} + \frac{\partial^2 \phi_1}{\partial \eta^2} + \frac{\partial^2 \phi_1}{\partial \zeta^2} = \alpha_d \left(c_{d1}\phi_2 + c_{d2}\phi_1^2 + n_{d2} \right) \tag{50}$$

$$-(\lambda_{d0} - v_{d0}) \frac{\partial n_{d2}}{\partial \zeta} + \frac{\partial u_{d2}}{\partial \xi} + \frac{\partial v_{d2}}{\partial \eta} + \frac{\partial w_{d2}}{\partial \zeta} = -\frac{\partial n_{d1}}{\partial \tau} - \frac{\partial}{\partial \zeta} (n_{d1} w_{d1}) \tag{51}$$

Proceeding along the lines of previous calculations for dispersion relation , we obtain a dispersion relation for the DAS wave given by ZK-Burgers equation in the form

$$\frac{\partial \phi_1}{\partial \tau} + A' \phi_1 \frac{\partial \phi_1}{\partial \zeta} + B' \frac{\partial^3 \phi_1}{\partial \zeta^3} + C' \frac{\partial}{\partial \zeta} \left(\frac{\partial^2 \phi_1}{\partial \xi^2} + \frac{\partial^2 \phi_1}{\partial \eta^2} \right) - D' \left(\frac{\partial^2 \phi_1}{\partial \xi^2} + \frac{\partial^2 \phi_1}{\partial \eta^2} + \frac{\partial^2 \phi_1}{\partial \zeta^2} \right) - E' \frac{\partial^2 \phi_1}{\partial \zeta^2} = 0 \tag{52}$$

where

$$A' = \frac{\sigma_d c_{d1}^3 - 2c_2 - 3c_{d1}^3 \alpha_d (\lambda_{do} - v_{do})^2}{2\alpha_d c_{d1}^2 (\lambda_{do} - v_{do})} \tag{53}$$

$$B' = \frac{1}{2c_{d1}^2 \alpha_d^2 (\lambda_{do} - v_{do})} \tag{54}$$

$$C' = \frac{1 + \Omega_c^{-2} \alpha_d c_{d1} (c_{d1}\sigma_d + 1)(\lambda_{do} - v_{do})^2}{2\alpha_d^2 c_{d1}^2 (\lambda_{do} - v_{do})} \tag{55}$$

$$D' = \frac{\eta_0}{2}, \ E' = \frac{\eta_0 + \mu_0}{2} \tag{56}$$

Hence the Burger terms having coefficients (D' and E') which are responsible for the generation of shock wave, originates due to the viscosity term. Also note that since dissipative terms are proportional to η_0 and μ_0,it mean in the absence of this the dissipative terms would vanish and ZK-Burger equation will be reduced to usual ZK equation admitting nonlinear soliton solutions only.

3.4 Stationary solution and quantitative analysis

In order to investigate the temporal evolution profiles of large amplitude localized DA waves in dusty plasma with Maxwellian and non Maxwellian Kappa and Cairns dispersed pair ions and electrons, we numerically solve Eq. (23) in this section. For example, if the coefficients of the Burger term resulting from the viscous nature of plasma are positive, i.e. (D',E'>0), then we can study some typical numerical characteristics of the shock-like structures[30]. We present the transformation to get the solution of ZKB equation (61).

$$\chi = \gamma_x \xi + \gamma_y \eta + \gamma_z \zeta - U_d \tau \tag{57}$$

where $\gamma_{s=x,y,z}$ are the direction cosines. whereas U_d is now normalized to C_{sd}. Using Eq. (66) into Eq. (61), yields

$$-U_d \frac{d\phi_1}{d\chi} + A^{'} \gamma_z \phi_1 \frac{d\phi_1}{d\chi} + H^{'} \gamma_z \frac{d^3\phi_1}{d\chi^3} - G^{'} \frac{d^2\phi_1}{d\chi^2} = 0, \tag{58}$$

where $H^{'} = \gamma_z^2 B^{'} + \left(\gamma_x^2 + \gamma_y^2\right) C^{'}$ and $G^{'} = E^{'} \gamma_z^2 + D^{'}$. Again employing the hyperbolic tangent (tanh) method along with the boundary conditions we found the shock wave solutions

$$\phi_1(\chi) = \frac{3}{25} \frac{G^{'2}}{H^{'} A^{'} \gamma_z^2} \left[2 - 2 \tanh\left(\frac{G^{'}}{10 H^{'} \gamma_z} \chi \right) + \mathrm{sech}^2 \left(\frac{G^{'}}{10 H^{'} \gamma_z} \chi \right) \right] \tag{59}$$

with, $10H^{'}\gamma_z G^{'-1}$ providing the width and $\frac{9}{25} G^2 H^{-1} A \gamma_z^{-2}$ gives the amplitude of shock waves moving with speed U_d. The shock width and amplitude are dependent on the dispersive coefficients $B^{'}$ and $C^{'}$, dissipative coefficients $D^{'}$ and $E^{'}$, and direction cosines γ_x, γ_y, γ_z. The amplitude is also dependent on the nonlinear coefficient $A^{'}$. The dependence of these coefficients on various plasma parameter determines the shape of the shock profile.

Now we numerical solve the Eq. ()

$l_z = 0.9$, $T_i = 2$ eV, $T_e = 8$eV, $Z_d = 50$, $n_{do} = 1/cm^3$, $n_{io} = 6 \times 10^3/cm^3$, $n_{eo} = 2 \times 10^2/cm^3$, $\alpha_d = \frac{Z_d n_{do}}{n_{io}}$, $\sigma_d = \frac{T_d}{T_e Z_d}$, $\sigma_i = \frac{T_i}{T_e}$,$\delta = \frac{n_{io}}{n_{do} Z_d}$, $\mu = \frac{n_{eo}}{n_{do} Z_d}$, $U_o = 0.1$, $\eta_o = 0.10$, $v_o = 0.05$, $\kappa = 4$, $\omega_c = 0.4$

Linear dispersion relation

It is discovered that increasing the proportion of suprathermal electrons (i.e., lowering kappa) increases the amplitude of the shock. Additionally, it is noted that the amplitude stays positive across the whole kappa value range. The range of kappa values that are typically present in space plasmas is the one we have selected.

Effect of rotation

The amplitude of the soliton is not directly affected by the external magnetic field or the rotational frequency magnitude, but the width of these solitary waves is. It is demonstrated that the breadth of these solitary waves reduces with increasing cyclotron and rotation frequencies. This indicates that the solitary structures become more spiky due to the coupling impact of the external magnetic field and rotation. Moreover, rotation causes the soliton's energy to diminish, which in turn causes the soliton um's amplitude to decrease. Therefore, a decreasing soliton amplitude might be linked to a decreasing soliton energy, which could be brought on by ions and positron reflection from the electrostatic field created inside plasma. The wave particle exchange mechanism, which can be calculated using kinetic theory and is outside the purview of this paper, could be another cause of the soliton amplitude decline.

Rotation has an impact on the ZK equation's dispersive coefficient but has no influence on the nonlinearity coefficient. Put otherwise, the amplitude of the wave in question cannot be changed by rotation. It is helpful to demonstrate how the rotation frequency affects the structure of the nonlinear acoustic wave since different planets and objects in the solar system have different rotation frequencies. It is observed that the increasing rotation frequency mitigates the width of the solitary wave for both kappa and crain distribution

Rotation causes the solitary wave's width to grow, which in turn reduces its amplitude and slows down its speed. Physically, the reflection of ions from the electrostatic field created inside the plasma may be the cause of this decrease in soliton energy.[31]

As far as the authors are aware, this stationary shock solution has not been investigated for the non-Maxwellian rotating viscous dusty plasma system. We anticipate that the current results will be helpful for both the PK-4 and the experiments being developed to investigate rotating dusty plasmas.

Chapter 4

Shock waves with charge and size fluctuations

4.1 Physical assumptions

The following basic assumptions are made that will help to formulate the physical problem and to find the explicit final results:

1. The dusty, electronegatively magnetized plasma is limitless, homogeneous, and collisionless.The plasma consist of electrons with number density n_e, positive ions with number density n_+, negative ions with number density n_- and moveable negative dust grains with number density n_d. The grains of dust have a negative charge, and this charge changes over time The constant weak magnetic field $\mathbf{B}$ lies in the $x - z$ plane, making an angle θ with the $x - axis$, and the propagation vector is along the $x - axis$.

2. A plasma flow v_{o} is present far upstream in the direction of the wave's propagation. i.e. along the $x-$direction and the plasma is considered to be in its balanced state defined by $\phi = 0$, $n_e = n_{eo}$, $n_+ = n_{+o}$, $n_- = n_{-o}$, $n_d = n_{do}$, and $q_d = -z_d e$, so that the plasma is quasi-neutral

$$n_{eo} + z_d n_{do} + n_{-o} = n_{+o}. \tag{4.1}$$

As is well known, the electron gyroradius is far lower than the dust size under a weak

magnetic field, thus the dust charge change is negligible. Here, electrons move quickly in the direction of the external magnetic field toward the dust grain surface, which could result in a Boltzmann distribution for fast electrons charging a grain.

Electrons, positive ions, and negative ions are thought to be Boltzmann distributed while studying low frequency motion in magnetoplasma. Therefore, in this instance, the electron and ion number densities are provided by

$$\frac{n_e}{n_{oe}} = z\exp(\Phi); \frac{n_+}{n_{+o}} = \exp(\frac{-\Phi}{\sigma_+}); \frac{n_-}{n_{-o}} = \exp(\frac{\Phi}{\sigma_-}) \tag{4.2}$$

here, $\sigma_+ = T_+/T_e$ and $\sigma_- = T_-/T_e$, where T_+, T_e and T_- are the temperatures of electrons, positive ions and negative ions, respectively, and $\Phi = e\phi/T_e$ is the normalized plasma potential.

3. The radius of dust grain is $r_0 << \rho_{e,+,-}$ (the gyroradius of the electron, and both positive and negative ions), so that the nonmagnetized case formulas can be used to approximate the current[32].

$$I_e = -J_e \exp\left(\Phi + z(q-1)\right),$$

$$I_- = -J_- \exp\left(\frac{\Phi + z(q-1}{\sigma_-}\right),$$

and

$$I_+ = J_+\left[1 - \frac{z(q-1)}{\sigma_-}\right]\exp\left(-\frac{\Phi}{\sigma_+}\right), \tag{4.3}$$

where $J_s = \pi r_o^2 e n_{so}\sqrt{8T_s/m_s}$, and $m_s$and T_s are the mass and temperature of the $s-$th species, while $z = z_d e^2/4\pi\varepsilon_o r_o T_e$, where $4\pi\varepsilon_o r_o T_e$ is the capacitance of the spherical dust grain. Note that the dust charge $q_d = -z_d e + \delta q_d$, where δq_d is the fluctuating dust charge so that $q_d/z_d e = -1 + q$; with $q = \delta q_d/z_{d0}e$ (normalized in units of equilibrium dust charge).

5. The ratio of the dust oscillation frequency ω_{pd} $(= \sqrt{z_d^2 e^2 n_{d0}/\varepsilon_o m_d})$ to the dust charging frequency v_{ch} $(\omega_{ch} = \omega_{pd}/v_{ch})$ is finite. On the other hand, because of the weak magnetic field, the ratio of dust cyclotron frequency Ω_d to ω_{pd} is small but $\neq 0$. Later in this

chapter, we will show that these assumptions are justified in cometary dusty plasma, e.g., for comet Halley.

4.2 Basic equations

The external magnetic field is based on the presumptions mentioned in the previous section. $B = B_o \cos\theta\widehat{x} + B_o \sin\theta\widehat{z}$, and the relative dust fluid velocity $v_d(x) = v_{dx}\widehat{x} + v_{dy}\widehat{y} + v_{dz}\widehat{z}$. Therefore, the normalized equations govern the nonlinear dynamics of the low phase velocity DAW:

$$\frac{\partial n_d}{\partial t} + \frac{\partial\left(n_d v_{dx}\right)}{\partial t} = 0, \tag{4.4}$$

$$\frac{\partial v_{dx}}{\partial t} + v_{dx}\frac{\partial v_{dx}}{\partial x} = -(-1+q)[\alpha_d^{-1}\frac{\partial \Phi}{\partial x} - \omega_{cd}v_{dy}\sin\theta] - \frac{\gamma_d\sigma_d}{\alpha_d}n_d^{\gamma_d-2}\frac{\partial n_d}{\partial x}, \tag{4.5}$$

$$\frac{\partial v_{dy}}{\partial t} + v_{dx}\frac{\partial v_{dy}}{\partial y} = \omega_{cd}(-1+q)[v_{dz}\cos\theta - v_{dx}\sin\theta], \tag{4.6}$$

$$\frac{\partial v_{dz}}{\partial t} + v_{dx}\frac{\partial v_{dz}}{\partial x} = -\omega_{cd}(-1+q)(v_{dy}\cos\theta), \tag{4.7}$$

and the Poisson's equation

$$\gamma\frac{\partial^2 \Phi}{\partial x^2} = \delta_+ \exp(\Phi) + \delta_- \exp(\Phi/\sigma_-) - \exp(-\Phi/\sigma_+) - \Delta n_d(-1+q), \tag{4.8}$$

where $n_d = n_d/n_{do}$, $\delta_d = T_d/z_d T_e$, $\Delta = 1 - \delta_+ - \delta_-, \delta_+ = n_{eo}/n_{+o}$, $\delta_- = n_{-o}/n_{+o}$, $\alpha_d = z_d n_{do}/\gamma n_{+o}$, $\gamma = (\delta_+ + 1/\sigma_+ + \delta_-/\sigma_-)$, T_d is the temperature of dust and γ_d is the adiabatic index. The normalized charge variable q is determined by the charging equation (Tsintsadze & Tsintsadze [2007]):

$$\frac{\partial q_d}{\partial t} = \sum_{s=e,+,-} I_s,$$

which in normalized form reads as

$$\frac{\partial q}{\partial t} = \frac{\sigma_+\beta_{ch}\omega_{ch}}{\left(1+\sigma_+ + \gamma_2^-\right)}\left[\begin{array}{c}\left(1-\frac{zq}{z+\sigma_+}\right)\exp(-\frac{\Phi}{\sigma_+}) - A_+\exp(\Phi+zq) \\ -A_-\exp\left(\frac{\Phi+zq}{\sigma_-}\right)\end{array}\right], \tag{4.9}$$

where

$$\beta_{ch} = \frac{(z+\sigma_+)(1-\sigma_+ + \gamma_{\bar{2}})}{z\sigma_+(1+z+\sigma_+ + \gamma_{\bar{1}})},$$

$$\gamma_{\bar{1}} = \frac{(z+\sigma_+)(1-\sigma_-)}{\sigma_-}A_-; \gamma_{\bar{2}} = \frac{\sigma_+(1-\sigma_-)}{\sigma_-}A_-, \tag{4.10}$$

$$A_+ = \frac{\delta_- \exp(-z)}{(z+\sigma_+)}\sqrt{\frac{\sigma_+ m_+}{m_e}},$$

$$\nu_{ch} = \frac{r_0}{\sqrt{2\pi}}\frac{\omega_{p+}^2}{v_{t+}}\frac{(z+\sigma_+)(1+\sigma_+ + \gamma_{\bar{2}})}{z\sigma_+\beta}.$$

and

$$A_- = \frac{\delta_- \exp(-z/\sigma_-)}{(z+\sigma_+)}\sqrt{\frac{\sigma_+\sigma_- m_+}{m_-}}.$$

Equilibrium current balance equation $I_{e0} + I_{+0} + I_{-0} = 0$ yields $A_+ + A_- = 1$. In the above, the time t and space x scales are, respectively, normalized in units of ω_{pd} and the plasma Debye length

$$\lambda_D = \left(\lambda_{De}^{-2} + \lambda_{D+}^{-2} + \lambda_{D-}^{-2}\right)^{1/2}, \lambda_{Ds} = (\varepsilon_o T_s/n_{so}e^2)^{1/2},$$

is the sth species particle Debye length. The dust fluid velocity v_d is normalized in units of dust acoustic speed $c_d = (Z_d T_e \alpha_d/m_d)^{1/2}$.

4.3 Nonlinear analysis: KdV Burger equation

The reductive perturbation technique is used to investigate the small but finite amplitude nonlinear DAW by stretching the independent variables as

$$\xi = \epsilon(x - v_{ph}t); \qquad \tau = \epsilon^2 t, \tag{4.11}$$

where ϵ is a small ($0 < \epsilon < 1$) expansion parameter, show the strength of the nonlinearity, and v_{ph} is the normalized phase velocity associated linear dust-acoustic wave. Multiple-scale expansion is the foundation of the reductive perturbation theory's general ideas. The dependent variables in this instance are represented as power series in ■.:

$$F = F_o + \epsilon F_1 + \epsilon^2 F_2 + ...,$$

and

$$v_{dy} = \epsilon^{3/2} v_{dy}^{(1)} + \epsilon^{5/2} v_{dy}^{(2)} + ..., \tag{4.12}$$

where $F = n_d, v_{dx}, v_{dz}, \varphi, q$ and $F_o = 1$, for v_o and n_d, while for v_{dx}, v_{dz}, φ, and q, $F_o = 0$.

i,e;

$$n_d = 1 + \epsilon n_d^{(1)} + \epsilon^2 n_d^{(2)} + ...$$

$$v_{dx} = v_0 + \epsilon v_{dx}^{(1)} + \epsilon^2 v_{dx}^{(2)} + ...$$

$$v_{dz} = \epsilon v_{dz}^{(1)} + \epsilon^2 v_{dz}^{(2)} + ...$$

$$\Phi = \epsilon \Phi^{(1)} + \epsilon^2 \Phi^{(2)} + ...$$

$$q = \epsilon q^{(1)} + \epsilon^2 q^{(2)} + ...$$

Also, because of the assumption (v), for a consistent perturbation expansion, it can be assumed that

$$\omega_{cd} = \frac{\Omega_d}{\omega_{pd}} = O(\sqrt{\epsilon}). \tag{4.13}$$

Now using chain rule:

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial \zeta} \times \frac{\partial \zeta}{\partial x} = \frac{\partial}{\partial \zeta} \times \frac{\partial}{\partial x} \left[\epsilon(x - v_{ph} t)\right]$$

$$\frac{\partial}{\partial x} = \epsilon \frac{\partial}{\partial \zeta} \text{ or } \frac{\partial^2}{\partial x^2} = \epsilon^2 \frac{\partial^2}{\partial \zeta^2} \tag{i}$$

and

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial \zeta} \times \frac{\partial \zeta}{\partial t} + \frac{\partial}{\partial \tau} \times \frac{\partial \tau}{\partial t}$$

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial \zeta} \times \frac{\partial}{\partial t} \left[\epsilon(x - v_{ph} t)\right] + \frac{\partial}{\partial \tau} \times \frac{\partial}{\partial t} \left[\epsilon^2 t\right]$$

$$= \frac{\partial}{\partial \zeta}\left(-v_{ph}\epsilon\right) + \epsilon^{^2}\frac{\partial}{\partial \tau}$$

$$= -\epsilon v_{ph}\frac{\partial}{\partial \zeta} + \epsilon^{^2}\frac{\partial}{\partial \tau} \tag{ii}$$

4.4 will become:

$$\frac{\partial n_d}{\partial t} + \frac{\partial\left(n_d v_{dx}\right)}{\partial t} = 0$$

$$\left(-\epsilon v_{ph}\frac{\partial}{\partial \zeta} + \epsilon^{^2}\frac{\partial}{\partial \tau}\right)\left(1 + \epsilon n_d^{(1)} + \epsilon^2 n_d^{(2)} + ...\right) + \left(\epsilon\frac{\partial}{\partial \zeta}\right)\left(1 + \epsilon n_d^{(1)} + \epsilon^2 n_d^{(2)} + ...\right)\left(v_0 + \epsilon v_{dx}^{(1)} + \epsilon^2 v_{dx}^{(2)} + ...\right) = 0$$

Comparing ϵ^2 order terms:

$$-v_{ph}\frac{\partial n_d^{(1)}}{\partial \zeta} + \frac{\partial v_{dx}^{(1)}}{\partial \zeta} + v_0\frac{\partial n_d^{(1)}}{\partial \zeta} = 0$$

$$-\left(v_{ph} - v_0\right)\frac{\partial n_d^{(1)}}{\partial \zeta} = -\frac{\partial v_{dx}^{(1)}}{\partial \zeta}$$

$$-\Lambda\frac{\partial n_d^{(1)}}{\partial \zeta} = -\frac{\partial v_{dx}^{(1)}}{\partial \zeta}$$

$$-\Lambda n_d^{(1)} = v_{dx}^{(1)} \tag{a}$$

Comparing ϵ^3 order terms:

$$\frac{\partial n_d^{(1)}}{\partial \tau} - v_{ph}\frac{\partial n_d^{(2)}}{\partial \zeta} + \frac{\partial v_{dx}^{(2)}}{\partial \zeta} + \frac{\partial\left(n_d^{(1)} v_{dx}^{(1)}\right)}{\partial \zeta} + v_0\frac{\partial n_d^{(2)}}{\partial \zeta} = 0$$

$$\frac{\partial n_d^{(1)}}{\partial \tau} - \left(v_{ph} - v_0\right)\frac{\partial n_d^{(2)}}{\partial \zeta} + \frac{\partial\left(n_d^{(1)} v_{dx}^{(1)}\right)}{\partial \zeta} + \frac{\partial v_{dx}^{(2)}}{\partial \zeta} = 0$$

$$\frac{\partial n_d^{(1)}}{\partial \tau} + \frac{\partial \left(n_d^{(1)} v_{dx}^{(1)} \right)}{\partial \zeta} - \Lambda \frac{\partial n_d^{(2)}}{\partial \zeta} + \frac{\partial v_{dx}^{(2)}}{\partial \zeta} = 0$$

$$\frac{\partial n_d^{(1)}}{\partial \tau} + \frac{\partial \left(n_d^{(1)} v_{dx}^{(1)} \right)}{\partial \zeta} = \Lambda \frac{\partial n_d^{(2)}}{\partial \zeta} - \frac{\partial v_{dx}^{(2)}}{\partial \zeta}$$

$$\frac{\partial n_d^{(1)}}{\partial \tau} + \frac{\partial \left(n_d^{(1)} v_{dx}^{(1)} \right)}{\partial \zeta} = \frac{\partial}{\partial \zeta} \left(\Lambda n_d^{(2)} - v_{dx}^{(2)} \right) \quad (4.14)$$

4.5 will become:

$$\frac{\partial v_{dx}}{\partial t} + v_{dx} \frac{\partial v_{dx}}{\partial x} = -(-1+q)[\alpha_d^{-1} \frac{\partial \Phi}{\partial x} - \omega_{cd} v_{dy} \sin\theta] - \frac{\gamma_d \sigma_d}{\alpha_d} n_d^{\gamma_d - 2} \frac{\partial n_d}{\partial x}$$

$$\left(-\epsilon v_{ph} \frac{\partial}{\partial \zeta} + \epsilon^2 \frac{\partial}{\partial \tau} \right) \left(v_0 + \epsilon v_{dx}^{(1)} + \epsilon^2 v_{dx}^{(2)} + ... \right) + \left(v_0 + \epsilon v_{dx}^{(1)} + \epsilon^2 v_{dx}^{(2)} + ... \right) \left(\epsilon \frac{\partial}{\partial \zeta} \right) (\gamma_d - 2) \left(v_0 + \epsilon v_{dx}^{(1)} + \epsilon^2 v_{dx}^{(2)} +$$

Comparing ϵ^2 order terms:

$$-v_{ph} \frac{\partial v_{dx}^{(1)}}{\partial \zeta} + v_0 \frac{\partial v_{dx}^{(1)}}{\partial \zeta} = \alpha_d^{-1} \frac{\partial \Phi^{(1)}}{\partial \zeta} - \omega_{cd} \sin\theta v_{dy}^{(1)} - \frac{\gamma_d \sigma_d}{\alpha_d} \frac{\partial n_d^{(1)}}{\partial \zeta}$$

$$-\left(v_{ph} - v_0 \right) \frac{\partial v_{dx}^{(1)}}{\partial \zeta} = \alpha_d^{-1} \frac{\partial}{\partial \zeta} \left[\Phi^{(1)} - \gamma_d \sigma_d n_d^{(1)} \right] - \omega_{cd} \sin\theta v_{dy}^{(1)}$$

$$-\Lambda \frac{\partial v_{dx}^{(1)}}{\partial \zeta} = \alpha_d^{-1} \frac{\partial}{\partial \zeta} \left[\Phi^{(1)} - \gamma_d \sigma_d n_d^{(1)} \right] - \omega_{cd} \sin\theta v_{dy}^{(1)} \quad \text{(b)}$$

Comparing ϵ^3 order terms:

$$\frac{\partial v_{dx}^{(1)}}{\partial \tau} - v_{ph} \frac{\partial v_{dx}^{(2)}}{\partial \zeta} + v_0 \frac{\partial v_{dx}^{(2)}}{\partial \zeta} + v_{dx}^{(1)} \frac{\partial v_{dx}^{(1)}}{\partial \zeta} = -\alpha_d^{-1} q^{(1)} \frac{\partial \Phi^{(1)}}{\partial \zeta} + \alpha_d^{-1} \frac{\partial \Phi^{(2)}}{\partial \zeta} + \omega_{cd} \sin\theta q^{(1)} v_{dy}^{(1)} - \omega_{cd} \sin\theta v_{dy}^{(2)} - \frac{\gamma_d \sigma_d}{\alpha_d} (\gamma_d - 2$$

$$\frac{\partial v_{dx}^{(1)}}{\partial \tau}-(v_{ph}-v_0)\frac{\partial v_{dx}^{(2)}}{\partial \zeta}+v_{dx}^{(1)}\frac{\partial v_{dx}^{(1)}}{\partial \zeta}=-\alpha_d^{-1}\left[q^{(1)}\frac{\partial \Phi^{(1)}}{\partial \zeta}+\gamma_d\sigma_d\left(\gamma_d-2\right)n_d^{(1)}\frac{\partial n_d^{(1)}}{\partial \zeta}\right]+\alpha_d^{-1}\frac{\partial}{\partial \zeta}\left[\Phi^{(2)}-\gamma_d\sigma_d\left(\gamma_d-2\right.\right.$$

$$\frac{\partial v_{dx}^{(1)}}{\partial \tau}+v_{dx}^{(1)}\frac{\partial v_{dx}^{(1)}}{\partial \zeta}+\alpha_d^{-1}\left[q^{(1)}\frac{\partial \Phi^{(1)}}{\partial \zeta}+\gamma_d\sigma_d\left(\gamma_d-2\right)n_d^{(1)}\frac{\partial n_d^{(1)}}{\partial \zeta}\right]=\alpha_d^{-1}\frac{\partial}{\partial \zeta}\left[\Phi^{(2)}-\gamma_d\sigma_d\left(\gamma_d-2\right)n_d^{(2)}\right]+\Lambda\frac{\partial v_{dx}^{(2)}}{\partial \zeta}+\omega_{c}\ \ (4.15)$$

4.6 will become:

$$\frac{\partial v_{dy}}{\partial t}+v_{dx}\frac{\partial v_{dy}}{\partial x}=\omega_{cd}(-1+q)[v_{dz}\cos\theta-v_{dx}\sin\theta]$$

$$\left(-\epsilon v_{ph}\frac{\partial}{\partial \zeta}+\epsilon^2\frac{\partial}{\partial \tau}\right)\left(\epsilon^{3/2}v_{dy}^{(1)}+\epsilon^{5/2}v_{dy}^{(2)}+...\right)+\left(v_0+\epsilon v_{dx}^{(1)}+\epsilon^2 v_{dx}^{(2)}+...\right)\left(\epsilon\frac{\partial}{\partial \zeta}\right)\left(\epsilon^{3/2}v_{dy}^{(1)}+\epsilon^{5/2}v_{dy}^{(2)}+...\right)=\omega$$

Comparing ϵ^1 order terms:

$$0=\omega_{cd}\left[-v_{dz}^{(1)}\cos\theta-v_{dx}^{(1)}\sin\theta\right]$$

$$v_{dz}^{(1)}\cos\theta=v_{dx}^{(1)}\sin\theta$$

$$v_{dz}^{(1)}=\frac{v_{dx}^{(1)}\sin\theta}{\cos\theta}$$

$$v_{dz}^{(1)}=\tan\theta v_{dx}^{(1)} \quad \text{(iii)}$$

Comparing ϵ^2 oreder terms:

$$-v_{ph}\frac{\partial v_{dy}^{(1)}}{\partial \zeta}+v_0\frac{\partial v_{dy}^{(1)}}{\partial \zeta}=\omega_{cd}\left[\left(q^{(1)}v_{dz}^{(1)}-v_{dz}^{(2)}\right)\cos\theta\right]-\omega_{cd}\left[\left(q^{(1)}v_{dx}^{(1)}-v_{dx}^{(2)}\right)\sin\theta\right]$$

$$-\Lambda \frac{\partial v_{dy}^{(1)}}{\partial \zeta} = \omega_{cd} q^{(1)} \left[v_{dz}^{(1)} \cos\theta - v_{dx}^{(1)} \sin\theta \right] - \omega_{cd} \left[v_{dz}^{(2)} \cos\theta - v_{dx}^{(2)} \sin\theta \right]$$

$$\Lambda \frac{\partial v_{dy}^{(1)}}{\partial \zeta} + \omega_{cd} q^{(1)} \left[-v_{dz}^{(1)} \cos\theta + v_{dx}^{(1)} \sin\theta \right] = \omega_{cd} \left[-v_{dz}^{(2)} \cos\theta + v_{dx}^{(2)} \sin\theta \right] \tag{4.16}$$

4.7 will become:

$$\frac{\partial v_{dz}}{\partial t} + v_{dx} \frac{\partial v_{dz}}{\partial x} = -\omega_{cd}(-1+q)(v_{dy}\cos\theta)$$

$$\left(-\epsilon v_{ph} \frac{\partial}{\partial \zeta} + \epsilon^{2} \frac{\partial}{\partial \tau} \right) \left(\epsilon v_{dz}^{(1)} + \epsilon^2 v_{dz}^{(2)} + ... \right) + \left(v_0 + \epsilon v_{dx}^{(1)} + \epsilon^2 v_{dx}^{(2)} + ... \right) \left(\epsilon \frac{\partial}{\partial \zeta} \right) \left(\epsilon v_{dz}^{(1)} + \epsilon^2 v_{dz}^{(2)} + ... \right) = -\omega_{cd} \left[-1 +$$

Comparing ϵ^2 order terms:

$$-v_{ph} \frac{\partial v_{dz}^{(1)}}{\partial \zeta} + v_0 \frac{\partial v_{dz}^{(1)}}{\partial \zeta} = -\omega_{cd} \left(-v_{dy}^{(1)} \cos\theta \right)$$

$$-\left(v_{ph} - v_0 \right) \frac{\partial v_{dz}^{(1)}}{\partial \zeta} = \omega_{cd} v_{dy}^{(1)} \cos\theta$$

$$-\Lambda \frac{\partial v_{dz}^{(1)}}{\partial \zeta} = \omega_{cd} v_{dy}^{(1)} \cos\theta \tag{c}$$

Comparing ϵ^3 order terms:

$$\frac{\partial v_{dz}^{(1)}}{\partial \tau} - v_{ph} \frac{\partial v_{dz}^{(2)}}{\partial \zeta} + v_{dx}^{(1)} \frac{\partial v_{dz}^{(2)}}{\partial \zeta} + v_0 \frac{\partial v_{dz}^{(2)}}{\partial \zeta} = -\omega_{cd} \left[q^{(1)} v_{dy}^{(1)} \cos\theta - v_{dy}^{(2)} \cos\theta \right]$$

$$\frac{\partial v_{dz}^{(1)}}{\partial \tau} + v_{dx}^{(1)} \frac{\partial v_{dz}^{(2)}}{\partial \zeta} - \left(v_{ph} - v_0 \right) \frac{\partial v_{dz}^{(2)}}{\partial \zeta} = -\omega_{cd} \left[q^{(1)} v_{dy}^{(1)} - v_{dy}^{(2)} \right] \cos\theta$$

$$\frac{\partial v_{dz}^{(1)}}{\partial \tau} + v_{dx}^{(1)} \frac{\partial v_{dz}^{(2)}}{\partial \zeta} = \Lambda \frac{\partial v_{dz}^{(2)}}{\partial \zeta} - -\omega_{cd} \left[q^{(1)} v_{dy}^{(1)} - v_{dy}^{(2)} \right] \cos\theta \tag{4.17}$$

4.8 will become:

$$\gamma\frac{\partial^2\Phi}{\partial x^2} = \delta_+\exp(\Phi) + \delta_-\exp(\Phi/\sigma_-) - \exp(-\Phi/\sigma_+) - \Delta n_d(-1+q)$$

$$\gamma\frac{\partial^2\Phi}{\partial x^2} = \delta_+\left[1+\Phi+\frac{\Phi^2}{2!}+...\right]+\delta_-\left[1+\frac{\Phi}{\sigma_-}+\frac{\Phi^2}{2!\sigma_-^2}+...\right]-\left[1-\frac{\Phi}{\sigma_+}+\frac{\Phi^2}{2!\sigma_+^2}+...\right]-(1-\delta_+-\delta_-)\,n_d(-1+q)$$

$$\gamma\left(\epsilon^2\frac{\partial^2}{\partial\zeta^2}\right)\left(\epsilon\Phi^{(1)}+\epsilon^2\Phi^{(2)}+...\right) = \delta_+\left[1+\left(\epsilon\Phi^{(1)}+\epsilon^2\Phi^{(2)}+...\right)+\frac{\left(\epsilon\Phi^{(1)}+\epsilon^2\Phi^{(2)}+...\right)^2}{2!}+...\right]-\delta_-\left[1+\frac{(\epsilon\Phi}{}\right.$$

Comparing ϵ^1 order terms:

$$0 = \delta_+\Phi^{(1)}+\frac{\delta_-}{\sigma_-}\Phi^{(1)}+\frac{1}{\sigma_+}\Phi^{(1)}-(1-\delta_+-\delta_-)\left[q^{(1)}-n_d^{(2)}\right]$$

$$0 = \left(\delta_++\frac{\delta_-}{\sigma_-}+\frac{1}{\sigma_+}\right)\Phi^{(1)}-(1-\delta_+-\delta_-)\left[Q^{q(1)}-n_d^{(2)}\right] \tag{d}$$

Comparing ϵ^2 order terms:

$$0 = \frac{1}{\gamma}\left[\delta_+\left(\Phi^{(2)}+\frac{\Phi^{(1)2}}{2}\right)+\delta_-\left(\frac{\Phi^{(2)}}{\sigma_-}+\frac{\Phi^{(1)2}}{2\sigma_-^2}\right)+\left(\frac{\Phi^{(2)}}{\sigma_+}+\frac{\Phi^{(1)2}}{2\sigma_+^2}\right)-(1-\delta_+-\delta_-)\left(n_d^{(1)}q^{(1)}-n_d^{(2)}+q^{(2)}\right)\right]$$

$$0 = \frac{1}{\gamma}\left[\left(\delta_++\frac{\delta_-}{\sigma_-}+\frac{1}{\sigma_+}\right)\Phi^{(2)}\right]+\frac{1}{2\gamma}\left[\delta_++\frac{\delta_-}{\sigma_-}+\frac{1}{\sigma_+^2}\right]\Phi^{(1)2}-\frac{(1-\delta_+-\delta_-)}{\gamma}\left[q^{(2)}-n_d^{(2)}+n_d^{(1)}q^{(1)}\right]$$

$$0 = \frac{1}{\gamma}(\gamma)\,\Phi^{(2)}+\frac{1}{2\gamma}\left[\delta_++\frac{\delta_-}{\sigma_-}+\frac{1}{\sigma_+^2}\right]\Phi^{(1)2}-\frac{(1-\delta_+-\delta_-)}{\gamma}\left[q^{(2)}-n_d^{(2)}+n_d^{(1)}q^{(1)}\right]$$

$$\Phi^{(2)}+\frac{1}{2\gamma}\left[\delta_++\frac{\delta_-}{\sigma_-}+\frac{1}{\sigma_+^2}\right]\Phi^{(1)2}-\alpha_d\left[q^{(2)}-n_d^{(2)}+n_d^{(1)}q^{(1)}\right]=0 \tag{4.18}$$

4.9 will become:

$$\frac{\partial q}{\partial t} = \frac{\sigma_+ \beta_{ch} \omega_{ch}}{\left(1+\sigma_+ + \gamma_2^-\right)} \left[\begin{array}{c} \left(1 - \frac{zq}{z+\sigma_+}\right) \exp(-\frac{\Phi}{\sigma_+}) - A_+ \exp(\Phi + zq) \\ -A_- \exp\left(\frac{\Phi+zq}{\sigma_-}\right) \end{array} \right]$$

$$\omega_{ch} \frac{\partial q}{\partial t} = \frac{\sigma_+ \beta_{ch}}{\left(1+\sigma_+ + \gamma_2^-\right)} \left[\left(1 - \frac{zq}{z+\sigma_+}\right) \left(1 - \left(\frac{\Phi}{\sigma_+}\right) + \left(\frac{\Phi^2}{2!\sigma_+^2}\right) + ...\right) - A_+ \left\{ 1 + (\Phi + zq) + \left(\frac{(\Phi+zq)^2}{2!}\right) + \right.\right.$$

$$\omega_{ch}\left(-\epsilon v_{ph} \frac{\partial}{\partial \zeta} + \epsilon^2 \frac{\partial}{\partial \tau}\right)\left(\epsilon q^{(1)} + \epsilon^2 q^{(2)} + ...\right) = \frac{\sigma_+ \beta_{ch}}{\left(1+\sigma_+ + \gamma_2^-\right)} \left[\left(1 - \frac{z\left(\epsilon q^{(1)} + \epsilon^2 q^{(2)} + ...\right)}{z+\sigma_+}\right) \left(1 - \frac{\left(\epsilon \Phi^{(1)} + \epsilon^2\right.}{\sigma}\right.\right.$$

Comparing ϵ^1 oreder terms:

$$0 = \frac{\sigma_+ \beta_{ch}}{\left(1+\sigma_+ + \gamma_2^-\right)} \left[-\left(\frac{\Phi^{(1)}}{\sigma_+} + \frac{zq^{(1)}}{z+\sigma_+}\right) - A_+ \left(\Phi^{(1)} + zq^{(1)}\right) - A_- \left(\frac{\Phi^{(1)} + zq^{(1)}}{\sigma_-}\right)\right] \qquad \text{(e)}$$

Comparing ϵ^2 order terms:

$$\omega_{ch}\left(-v_{ph}\right) \frac{\partial q^{(1)}}{\partial \zeta} = \frac{\sigma_+ \beta_{ch}}{\left(1+\sigma_+ + \gamma_2^-\right)} \left[\left(-\frac{\Phi^{(2)}}{\sigma_+} - \frac{zq^{(2)}}{z+\sigma_+} + \frac{zq^{(1)}\Phi^{(1)}}{\sigma_+\left(z+\sigma_+\right)} + \frac{{\Phi^{(1)}}^2}{2\sigma_+^2}\right) - A_+ \left(\Phi^{(2)} + zq^{(2)} + \frac{{\Phi^{(1)}}^2}{2} + \right.\right.$$

$$-\Lambda \omega_{ch} \frac{\partial q^{(1)}}{\partial \zeta} = \frac{\sigma_+ \beta_{ch}}{\left(1+\sigma_+ + \gamma_2^-\right)} \left[\left(-\frac{1}{\sigma_+} - A_+ - \frac{A_-}{\sigma_-}\right)\Phi^{(2)} - \left(\frac{z}{z+\sigma_+} + zA_+ + \frac{zA_-}{\sigma_-}\right) q^{(2)}\right] + \frac{\sigma_+ \beta_{ch}}{\left(1+\sigma_+ + \gamma_2^-\right)} \left[\right.$$

$$-\Lambda \omega_{ch} \frac{\partial q^{(1)}}{\partial \zeta} = \frac{\sigma_+ \beta_{ch}}{\left(1+\sigma_+ + \gamma_2^-\right)} \left[\left(-\frac{1}{\sigma_+} - A_+ - \frac{A_-}{\sigma_-}\right)\Phi^{(2)}\right] - \frac{\sigma_+ \beta_{ch}}{\left(1+\sigma_+ + \gamma_2^-\right)} \left[\left(\frac{z}{z+\sigma_+} + zA_+ + \frac{zA_-}{\sigma_-}\right) q^{(2)}\right] +$$

Now (1) will give:

$$\frac{\sigma_+\beta_{ch}}{\left(1+\sigma_+ +\gamma_2^-\right)}\left[\left(-\frac{1}{\sigma_+}-A_+-\frac{A_-}{\sigma_-}\right)\Phi^{(2)}\right]$$

$$=-\frac{\sigma_+\beta_{ch}}{\left(1+\sigma_+ +\frac{\sigma_+\left(1-\sigma_-\right)}{\sigma_-}A_-\right)}\left[\frac{1}{\sigma_+}+A_+ +\frac{A_-}{\sigma_-}\right]\Phi^{(2)}$$

$$=-\frac{\sigma_-\sigma_+\beta_{ch}}{\sigma_- +\sigma_-\sigma_+ +\sigma_+A_- -\sigma_-\sigma_+A_-}\left[\frac{\sigma_- +\sigma_-\sigma_+A_+ +\sigma_+A_-}{\sigma_-\sigma_+}\right]\Phi^{(2)}$$

$$=-\frac{\beta_{ch}}{\sigma_- +\sigma_-\sigma_+A_+ +\sigma_+A_-}\left[\sigma_- +\sigma_-\sigma_+A_+ +\sigma_+A_-\right]\Phi^{(2)}$$

$$=-\beta_{ch}\Phi^{(2)}$$

Also (2) will become:

$$-\frac{\sigma_+\beta_{ch}z}{\left(1+\sigma_+ +\gamma_2^-\right)}\left[\frac{1}{z+\sigma_+}+A_+ +\frac{A_-}{\sigma_-}\right]q^{(2)}$$

$$=-\frac{\sigma_+z\left(z+\sigma_+\right)\left(1+\sigma_+ +\gamma_2^-\right)}{\left(1+\sigma_+ +\gamma_2^-\right)\left(1+z+\sigma_+ +\gamma_1^-\right)}\left[\frac{\sigma_- +\sigma_-A_+\left(z+\sigma_+\right)+A_-\left(z+\sigma_+\right)}{\sigma_-\left(z+\sigma_+\right)}\right]q^{(2)}$$

$$=-\frac{1}{\left(1+z+\sigma_+ +\gamma_1^-\right)\sigma_-}\left[\sigma_- +\sigma_-A_+z+\sigma_-A_+\sigma_+ +A_-z+A_-\sigma_+\right]q^{(2)}$$

$$=-\frac{\left[\sigma_- +\sigma_-A_+z+\sigma_-A_+\sigma_+ +A_-z+A_-\sigma_+\right]}{\sigma_-\left[1+z+\sigma_+ +\frac{\left(z+\sigma_+\right)\left(1-\sigma_-\right)}{\sigma_-}A_-\right]}q^{(2)}$$

$$=-\frac{\left[\sigma_- +\sigma_-A_+z+\sigma_-A_+\sigma_+ +A_-z+A_-\sigma_+\right]}{\left[\sigma_- +\sigma_-A_+z+\sigma_-A_+\sigma_+ +A_-z+A_-\sigma_+\right]}q^{(2)}$$

$$=-q^{(2)}$$

Also (3) will become:

$$-\frac{\sigma_+\beta_{ch}}{\left(1+\sigma_++\gamma_2^-\right)}\left[\frac{1}{\sigma_+^2}-A_+-\frac{A_-}{\sigma_-^2}\right]\Phi^{(1)^2}$$

$$=-\frac{\sigma_+\beta_{ch}}{\left(1+\sigma_++\gamma_2^-\right)}\left[\frac{1}{\sigma_+^2}-\frac{A_-}{\sigma_-^2}-(1-A_-)\right]\Phi^{(1)^2}$$

$$=-\frac{\sigma_+\beta_{ch}}{\left(1+\sigma_++\gamma_2^-\right)}\left[\frac{1}{\sigma_+^2}-\frac{A_-}{\sigma_-^2}-1+A_-\right]\Phi^{(1)^2}$$

$$=-\frac{\sigma_+\beta_{ch}}{\left(1+\sigma_++\gamma_2^-\right)}\left[\frac{1}{\sigma_+^2}-1-\left(\frac{A_-}{\sigma_-^2}-A_-\right)\right]\Phi^{(1)^2}$$

$$=-\frac{\sigma_+\beta_{ch}}{\left(1+\sigma_++\gamma_2^-\right)}\left[\frac{1}{\sigma_+^2}-1-\left(\frac{A_-\left(1-\sigma_-^2\right)}{\sigma_-^2}\right)\right]\Phi^{(1)^2}$$

$$=-\frac{\sigma_+\beta_{ch}}{\left(1+\sigma_++\gamma_2^-\right)}\left[\frac{1}{\sigma_+^2}-1-B_-\right]\Phi^{(1)^2}$$

$$=-\frac{\sigma_+\beta_{ch}}{\left(1+\sigma_++\gamma_2^-\right)}\left[\frac{1}{\sigma_+^2}-(1+B_-)\right]\Phi^{(1)^2}$$

$$=-\frac{\sigma_+\beta_{ch}}{\left(1+\sigma_++\gamma_2^-\right)}\left[\frac{1-\sigma_+^2\,(1+B_-)}{\sigma_+^2}\right]\Phi^{(1)^2}$$

Now (4) will become:

$$-\frac{\sigma_+\beta_{ch}}{\left(1+\sigma_++\gamma_2^-\right)}\left[\frac{z^2q^{(1)2}}{2}A_++\frac{z^2q^{(1)2}}{2\sigma_-^2}A_-\right]q^{(1)2}$$

$$=-\frac{\sigma_+\beta_{ch}z^2}{\left(1+\sigma_++\gamma_2^-\right)}\left[\frac{1}{2}A_++\frac{1}{2\sigma_-^2}A_-\right]q^{(1)2}$$

$$=-\frac{\sigma_+\beta_{ch}z^2}{2\left(1+\sigma_++\gamma_2^-\right)}\left[\frac{A_-}{\sigma_-^2}+A_+\right]q^{(1)2}$$

$$= -\frac{\sigma_+ \beta_{ch} z^2}{2\left(1+\sigma_+ + \gamma_2^-\right)} \left[\frac{A_-}{\sigma_-^2} + (1 - A_-)\right] q^{(1)2}$$

$$= -\frac{\sigma_+ \beta_{ch} z^2}{2\left(1+\sigma_+ + \gamma_2^-\right)} \left[\frac{A_- + \sigma_-^2 - \sigma_-^2 A_-}{\sigma_-^2}\right] q^{(1)2}$$

$$= -\frac{\sigma_+ \beta_{ch} z^2}{2\left(1+\sigma_+ + \gamma_2^-\right)} \left[\frac{\sigma_-^2 + A_-\left(1 - \sigma_-^2\right)}{\sigma_-^2}\right] q^{(1)2}$$

$$= -\frac{\sigma_+ \beta_{ch} z^2}{2\left(1+\sigma_+ + \gamma_2^-\right)} \left[1 + \frac{A_-\left(1 - \sigma_-^2\right)}{\sigma_-^2}\right] q^{(1)2}$$

$$= -\frac{\sigma_+ \beta_{ch} z^2}{2\left(1+\sigma_+ + \gamma_2^-\right)} \left[1 + B_-\right] q^{(1)2}$$

where,

$$B_- = \frac{A_-\left(1 - \sigma_-^2\right)}{\sigma_-^2}$$

Now (5) will be:

$$-\frac{\sigma_+ \beta_{ch} z}{\left(1+\sigma_+ + \gamma_2^-\right)} \left[\frac{1}{\sigma_+\left(z + \sigma_+\right)} - A_+ - \frac{1}{\sigma_-^2} A_-\right] q^{(1)} \Phi^{(1)}$$

$$= -\frac{\sigma_+ \beta_{ch} z}{\left(1+\sigma_+ + \gamma_2^-\right)} \left[\frac{1}{\sigma_+\left(z + \sigma_+\right)} - (1 - A_-) - \frac{1}{\sigma_-^2} A_-\right] q^{(1)} \Phi^{(1)}$$

$$= -\frac{\sigma_+ \beta_{ch} z}{\left(1+\sigma_+ + \gamma_2^-\right)} \left[\frac{1}{\sigma_+\left(z + \sigma_+\right)} - 1 + A_- - \frac{1}{\sigma_-^2} A_-\right] q^{(1)} \Phi^{(1)}$$

$$= -\frac{\sigma_+ \beta_{ch} z}{\left(1+\sigma_+ + \gamma_2^-\right)} \left[\frac{1}{\sigma_+\left(z + \sigma_+\right)} - \left(1 - A_- + \frac{1}{\sigma_-^2} A_-\right)\right] q^{(1)} \Phi^{(1)}$$

$$= -\frac{\sigma_+ \beta_{ch} z}{\left(1+\sigma_+ + \gamma_2^-\right)} \left[\frac{1}{\sigma_+\left(z + \sigma_+\right)} - \frac{\sigma_-^2 - \sigma_-^2 A_- + A_-}{\sigma_-^2}\right] q^{(1)} \Phi^{(1)}$$

$$= -\frac{\sigma_+ \beta_{ch} z}{\left(1+\sigma_+ + \gamma_2^-\right)} \left[\frac{1}{\sigma_+ \left(z+\sigma_+\right)} - 1 - \frac{A_- \left(1-\sigma_-^2\right)}{\sigma_-^2}\right] q^{(1)} \Phi^{(1)}$$

$$= -\frac{\sigma_+ \beta_{ch} z}{\left(1+\sigma_+ + \gamma_2^-\right)} \left[\frac{1}{\sigma_+ \left(z+\sigma_+\right)} - 1 - B_-\right] q^{(1)} \Phi^{(1)}$$

$$= -\frac{\sigma_+ \beta_{ch} z}{\left(1+\sigma_+ + \gamma_2^-\right)} \left[\frac{1}{\sigma_+ \left(z+\sigma_+\right)} - \left(1 + B_-\right)\right] q^{(1)} \Phi^{(1)}$$

$$= -\frac{\sigma_+ \beta_{ch} z}{\left(1+\sigma_+ + \gamma_2^-\right)} \left[\frac{1 - \sigma_+ \left(z+\sigma_+\right)\left(1 + B_-\right)}{\sigma_+ \left(z+\sigma_+\right)}\right] q^{(1)} \Phi^{(1)}$$

So (a) will be :

$$-\Lambda \omega_{ch} \frac{\partial q^{(1)}}{\partial \zeta} = -\beta_{ch} \Phi^{(2)} - \left(-q^{(2)}\right) + \frac{\sigma_+ \beta_{ch}}{2\left(1+\sigma_+ + \gamma_2^-\right)} \left\{ \left(\frac{1-\sigma_+^2 \left(1+B_-\right)}{\sigma_+^2}\right) \Phi^{(1)^2} - z^2 \left(1+B_-\right) q^{(1)2} + z \left(\frac{1-\sigma \ldots\right.\right.$$

$$q^{(2)} + \beta_{ch} \Phi^{(2)} + \Lambda \omega_{ch} \frac{\partial q^{(1)}}{\partial \zeta} + \frac{\sigma_+ \beta_{ch}}{2\left(1+\sigma_+ + \gamma_2^-\right)} \left[z^2 \left(1+B_-\right) q^{(1)2} - \left(\frac{1-\sigma_+^2 \left(1+B_-\right)}{\sigma_+^2}\right) \Phi^{(1)^2} - z \left(\frac{1-\sigma_+ \left(z+\sigma_+ \ldots\right.}{\sigma_+ \left(z+ \ldots\right.}\right.\right. \tag{4.19}$$

(c) will give:

$$-\Lambda \frac{\partial v_{dz}^{(1)}}{\partial \zeta} = \omega_{cd} v_{dy}^{(1)} \cos\theta$$

$$\Lambda \frac{\partial}{\partial \zeta} \left(\tan\theta v_{dx}^{(1)}\right) = -\omega_{cd} v_{dy}^{(1)} \cos\theta$$

$$\Lambda \tan\theta \frac{\partial v_{dx}^{(1)}}{\partial \zeta} = -\omega_{cd} v_{dy}^{(1)} \cos\theta$$

$$\Lambda \tan\theta \frac{\partial}{\partial \zeta} \left(\Lambda n_d^{(1)}\right) = -\omega_{cd} v_{dy}^{(1)} \cos\theta$$

$$\Lambda^2 \tan\theta \frac{\partial n_d^{(1)}}{\partial \zeta} = -\omega_{cd} v_{dy}^{(1)} \cos\theta$$

$$v_{dy}^{(1)} = -\frac{\Lambda^2 \sin\theta \sec^2\theta}{\omega_{cd}} \frac{\partial n_d^{(1)}}{\partial \zeta} \tag{v}$$

(b) will give:

$$-\Lambda \frac{\partial v_{dx}^{(1)}}{\partial \zeta} = \alpha_d^{-1} \frac{\partial}{\partial \zeta} \left[\Phi^{(1)} - \gamma_d \sigma_d n_d^{(1)}\right] - \omega_{cd} \sin\theta v_{dy}^{(1)}$$

$$-\Lambda \frac{\partial \left(\Lambda n_d^{(1)}\right)}{\partial \zeta} = \alpha_d^{-1} \frac{\partial \Phi^{(1)}}{\partial \zeta} - \frac{\gamma_d \sigma_d}{\alpha_d} \frac{\partial n_d^{(1)}}{\partial \zeta} - \omega_{cd} \sin\theta v_{dy}^{(1)}$$

$$-\Lambda^2 \frac{\partial n_d^{(1)}}{\partial \zeta} + \frac{\gamma_d \sigma_d}{\alpha_d} \frac{\partial n_d^{(1)}}{\partial \zeta} - \omega_{cd} \sin\theta \left(-\frac{\Lambda^2 \sin\theta \sec^2\theta}{\omega_{cd}} \frac{\partial n_d^{(1)}}{\partial \zeta}\right) = \frac{1}{\alpha_d} \frac{\partial \Phi^{(1)}}{\partial \zeta}$$

$$n_d^{(1)} \left(-\Lambda^2 + \frac{\gamma_d \sigma_d}{\alpha_d} - \Lambda^2 \sin^2\theta \sec^2\theta\right) = \frac{1}{\alpha_d} \Phi^{(1)}$$

$$n_d^{(1)} \left[-\Lambda^2 \left(1 + \sin^2\theta \sec^2\theta\right) + \frac{\gamma_d \sigma_d}{\alpha_d}\right] = \frac{1}{\alpha_d} \Phi^{(1)}$$

$$n_d^{(1)} \left[-\Lambda^2 \left(1 + \tan^2\theta\right) + \frac{\gamma_d \sigma_d}{\alpha_d}\right] = \frac{1}{\alpha_d} \Phi^{(1)}$$

$$n_d^{(1)} \left[-\Lambda^2 \sec^2\theta + \frac{\gamma_d \sigma_d}{\alpha_d}\right] = \frac{1}{\alpha_d} \Phi^{(1)}$$

$$\Phi^{(1)} = \alpha_d n_d^{(1)} \left[-\Lambda^2 \sec^2\theta + \frac{\gamma_d \sigma_d}{\alpha_d}\right] \tag{vi}$$

Also,

$$\Phi^{(1)} = -\frac{q^{(1)}}{\beta_{ch}} \tag{vi}$$

By comparing, we get:

$$q^{(1)} = -\beta_{ch}\alpha_d\left[-\Lambda^2\sec^2\theta + \frac{\gamma_d\sigma_d}{\alpha_d}\right]n_d^{(1)}$$

$$q^{(1)} = \beta_{ch}\alpha_d\left[\Lambda^2\sec^2\theta - \frac{\gamma_d\sigma_d}{\alpha_d}\right]n_d^{(1)} \tag{vii}$$

Also

$$q^{(1)} = \left[1 - \Lambda^2\sec^2\theta + \frac{\gamma_d\sigma_d}{\alpha_d}\right]n_d^{(1)} \tag{viii}$$

Comparing (vii) & (viii),we get:

$$\left[1 - \Lambda^2\sec^2\theta + \frac{\gamma_d\sigma_d}{\alpha_d}\right]n_d^{(1)} = \beta_{ch}\alpha_d\left[\Lambda^2\sec^2\theta - \frac{\gamma_d\sigma_d}{\alpha_d}\right]n_d^{(1)}$$

$$1 - \Lambda^2\sec^2\theta + \frac{\gamma_d\sigma_d}{\alpha_d} = \beta_{ch}\alpha_d\left(\Lambda^2\sec^2\theta - \frac{\gamma_d\sigma_d}{\alpha_d}\right)$$

$$1 + \frac{\gamma_d\sigma_d}{\alpha_d} + \alpha_d\beta_{ch}\left(\frac{\gamma_d\sigma_d}{\alpha_d}\right) = \alpha_d\beta_{ch}\Lambda^2\sec^2\theta + \Lambda^2\sec^2\theta$$

$$1 + \frac{\gamma_d\sigma_d}{\alpha_d}\left(1 + \alpha_d\beta_{ch}\right) = \left(\alpha_d\beta_{ch} + 1\right)\Lambda^2\sec^2\theta$$

$$\frac{1}{(1 + \alpha_d\beta_{ch})} + \frac{\gamma_d\sigma_d}{\alpha_d} = \Lambda^2\sec^2\theta$$

$$\Lambda^2 = \cos^2\theta\left(\frac{\gamma_d\sigma_d}{\alpha_d} + \frac{1}{(1 + \alpha_d\beta_{ch})}\right)$$

$$\Lambda = \cos\theta\left(\frac{\gamma_d\sigma_d}{\alpha_d} + \frac{1}{(1 + \alpha_d\beta_{ch})}\right)^{1/2}$$

and

$$\Lambda = v_0 - v_{ph}$$

$$v_{ph} = -\Lambda + v_0$$

$$v_{ph} = v_0 - \cos\theta \left(\frac{\gamma_d \sigma_d}{\alpha_d} + \frac{1}{(1+\alpha_d \beta_{ch})} \right)^{1/2}$$

$$v_{ph} \simeq \cos\theta \left(\frac{\gamma_d \sigma_d}{\alpha_d} + \frac{1}{(1+\alpha_d \beta_{ch})} \right)^{1/2} \tag{4.20}$$

Hence equating to lowest powers of ϵ, the following relations are obtained:

$$v_{dz}^{(1)} = \tan\theta v_{dx}^{(1)}, \tag{4.21}$$

$$q^{(1)} = \left(1 - \Lambda^2 \sec^2\theta + \frac{\gamma_d \sigma_d}{\alpha_d} \right) n_d^{(1)},$$

and

$$v_{dy}^{(1)} = -\frac{\Lambda^2 \sin\theta \sec^2\theta}{\omega_{cd}} \frac{\partial n_d^{(1)}}{\partial \xi}, \tag{4.22}$$

where $\Lambda = v_{ph} - v_o$. The above set of equations (6.14) self-consistently yields the following linear wave phase velocity

$$v_{ph} = \mathbf{v}_o - \cos\theta \left(\frac{\gamma_d \sigma_d}{\alpha_d} + \frac{1}{1+\alpha_d \beta_{ch}} \right)^{1/2}. \tag{4.23}$$

The terms of the subsequent higher powers of ■ are then equated to get the following relations:

$$\frac{\partial n_d^{(1)}}{\partial \tau} + \frac{\partial (n_d^{(1)} v_{dx}^{(1)})}{\partial \xi} = -\frac{\partial (\Lambda n_d^{(2)} + u_{dx}^{(2)})}{\partial \xi}, \tag{I}$$

$$\frac{\partial v_{dx}^{(1)}}{\partial \tau} + v_{dx}^{(1)} \frac{\partial v_{dx}^{(1)}}{\partial \xi} + \frac{1}{\alpha_d} \left(q^{(1)} \frac{\partial \varphi^{(1)}}{\partial \xi} + \gamma_d (\gamma_d - 2) \sigma_d n_d^{(1)} \frac{\partial n_d^{(1)}}{\partial \xi} \right) \tag{II}$$

$$= \frac{1}{\alpha_d} \left(\frac{\partial (\varphi^{(2)} - \gamma_d \alpha_d n_d^{(2)})}{\partial \xi} - \Lambda \frac{\partial v_{dx}^{(2)}}{\partial \xi} \right) + \omega_{cd} (q^{(1)} v_{dx}^{(1)} - v_{dy}^{(2)}) \sin\theta, \tag{4.1}$$

$$\Lambda\frac{\partial v_{dy}^{(1)}}{\partial\xi}+\omega_{cd}q^{(1)}(v_{dx}^{(1)}\sin\theta-v_{dz}^{(1)}\cos\theta)=\omega_{cd}q^{(1)}(v_{dx}^{(2)}\sin\theta-v_{dz}^{(2)}\cos\theta), \tag{III}$$

$$\frac{\partial v_{dz}^{(1)}}{\partial\tau}+v_{dx}^{(1)}\frac{\partial v_{dz}^{(1)}}{\partial\xi}=-\Lambda\frac{\partial v_{dz}^{(2)}}{\partial\xi}-\omega_{cd}(q^{(1)}v_{dy}^{(1)}-v_{dz}^{(2)})\cos\theta, \tag{IV}$$

$$\varphi^{(2)}+\frac{1}{2\gamma}\left(\delta_{+}+\frac{\delta_{-}}{\sigma_{-}^{2}}-\frac{1}{\sigma_{+}^{2}}\right)\varphi^{(1)^2}-\alpha_d(q^{(2)}-n_d^{(2)}+q^{(1)}n_d^{(1)})=0, \tag{V}$$

and

$$q^{(2)}+\beta_{ch}\varphi^{(2)}+\Lambda\omega_{ch}\frac{\partial q^{(1)}}{\partial\xi}+\frac{(\sigma_{+}\beta_{ch})z^2(1+B_{-})q^{(1)^2}}{2(1+\sigma_{+}+\gamma_{\overline{2}})}-\frac{1+\sigma_{+}^{2}(1+B_{-}))}{\sigma_{+}^{2}}\varphi^{(1)^2} \tag{VI}$$

$$-\frac{z(1-\sigma_{+}(z+\sigma_{+})(1+B_{-}))}{\sigma_{+}(z+\sigma_{+})}q^{(1)}\varphi^{(1)}=0, \tag{4.2}$$

where $B_{-}=(1-\sigma_{-}^{2})A_{-}/\sigma_{-}^{2}$.

Finally, eliminating all the second order quantities from Eqs. (I)-(VI),we get

$$\frac{\partial n_d^{(1)}}{\partial\tau}+\frac{\partial(n_d^{(1)}v_{dx}^{(1)})}{\partial\xi}=0 \tag{VII}$$

$$\frac{\partial v_{dx}^{(1)}}{\partial\tau}+v_{dx}^{(1)}\frac{\partial v_{dx}^{(1)}}{\partial\xi}+\frac{1}{\alpha_d}\left(q^{(1)}\frac{\partial\varphi^{(1)}}{\partial\xi}+\gamma_d(\gamma_d-2)\sigma_d n_d^{(1)}\frac{\partial n_d^{(1)}}{\partial\xi}\right)+\omega_{cd}q^{(1)}v_{dy}^{(1)}\sin\theta, \tag{VIII}$$

$$\Lambda\frac{\partial v_{dy}^{(1)}}{\partial\xi}+\omega_{cd}q^{(1)}(v_{dx}^{(1)}\sin\theta-v_{dz}^{(1)}\cos\theta)=0, \tag{IX}$$

$$\frac{\partial v_{dz}^{(1)}}{\partial\tau}+v_{dx}^{(1)}\frac{\partial v_{dz}^{(1)}}{\partial\xi}=-\omega_{cd}q^{(1)}v_{dy}^{(1)}\cos\theta, \tag{X}$$

$$\frac{1}{2\gamma}\left(\delta_{+}+\frac{\delta_{-}}{\sigma_{-}^{2}}-\frac{1}{\sigma_{+}^{2}}\right)\varphi^{(1)^2}-\alpha_d q^{(1)}n_d^{(1)}=0, \tag{XI}$$

$$\Lambda\omega_{ch}\frac{\partial q^{(1)}}{\partial\xi}+\frac{(\sigma_{+}\beta_{ch})z^2(1+B_{-})q^{(1)^2}}{2(1+\sigma_{+}+\gamma_{\overline{2}})}-\frac{1+\sigma_{+}^{2}(1+B_{-}))}{\sigma_{+}^{2}}\varphi^{(1)^2}-\frac{z(1-\sigma_{+}(z+\sigma_{+})(1+B_{-}))}{\sigma_{+}(z+\sigma_{+})}q^{(1)}\varphi^{(1)}=0, \tag{XII}$$

Now calculating (VII),

$$\frac{\partial n_d^{(1)}}{\partial \tau} + \frac{\partial (n_d^{(1)} v_{dx}^{(1)})}{\partial \xi} = 0$$

$$\frac{\partial n_d^{(1)}}{\partial \tau} + \frac{\partial}{\partial \xi}(n_d^{(1)} v_{dx}^{(1)}) = 0$$

$$\frac{\partial n_d^{(1)}}{\partial \tau} + \frac{\partial}{\partial \xi} n_d^{(1)} \left(-\Lambda n_d^{(1)}\right) = 0$$

$$\frac{\partial n_d^{(1)}}{\partial \tau} - \Lambda \frac{\partial}{\partial \xi} n_d^{2(1)} = 0$$

$$\frac{\partial n_d^{(1)}}{\partial \tau} - 2\Lambda n_d^{(1)} \frac{\partial}{\partial \xi} n_d^{(1)} = 0 \qquad (\text{VII}prime)$$

Now calculating (VIII),

$$\frac{\partial \left(-\Lambda n_d^{(1)}\right)}{\partial \tau} + \left(-\Lambda n_d^{(1)}\right) \frac{\partial \left(-\Lambda n_d^{(1)}\right)}{\partial \xi} + \frac{1}{\alpha_d} \left[\left(\frac{\alpha_d \beta_{ch}}{\alpha_d \beta_{ch} + 1} \right) n_d^{(1)} \frac{\partial}{\partial \xi} \left(\frac{-\alpha_d}{\alpha_d \beta_{ch} + 1} \right) n_d^{(1)} + \gamma_d(\gamma_d - 2)\sigma_d n_d^{(1)} \frac{\partial n_d^{(1)}}{\partial \xi} \right] =$$

$$-\Lambda \frac{\partial n_d^{(1)}}{\partial \tau} + \Lambda^2 n_d^{(1)} \frac{\partial n_d^{(1)}}{\partial \xi} + \frac{1}{\alpha_d} \left[\frac{\alpha_d^2}{(\alpha_d \beta_{ch} + 1)^2} \beta_{ch} n_d^{(1)} \frac{\partial n_d^{(1)}}{\partial \xi} + \gamma_d(\gamma_d - 2)\sigma_d n_d^{(1)} \frac{\partial n_d^{(1)}}{\partial \xi} \right] = -\frac{\Lambda^2 \sin^2\theta \sec^2\theta \alpha_d \beta_{ch}}{(\alpha_d \beta_{ch} + 1)} n_d^{(1)}$$

$$-\Lambda \frac{\partial n_d^{(1)}}{\partial \tau} + \left[\Lambda^2 - \frac{\alpha_d \beta_{ch}}{(\alpha_d \beta_{ch} + 1)^2} + \frac{\gamma_d(\gamma_d - 2)\sigma_d}{\alpha_d} + \frac{\Lambda^2 \sin^2\theta \sec^2\theta \alpha_d \beta_{ch}}{(\alpha_d \beta_{ch} + 1)} \right] n_d^{(1)} \frac{\partial n_d^{(1)}}{\partial \xi} = 0$$

Dividing by "$-\Lambda$",

$$\frac{\partial n_d^{(1)}}{\partial \tau} + \left[-\Lambda + \frac{\alpha_d \beta_{ch}}{\Lambda\left(\alpha_d \beta_{ch}+1\right)^2} - \frac{\gamma_d(\gamma_d-2)\sigma_d}{\Lambda \alpha_d} - \frac{\Lambda \sin^2\theta \sec^2\theta \alpha_d \beta_{ch}}{(\alpha_d \beta_{ch}+1)}\right] n_d^{(1)} \frac{\partial n_d^{(1)}}{\partial \xi} = 0 \quad (\mathrm{VIII}')$$

Calculating (IX),

$$\Lambda \frac{\partial v_{dy}^{(1)}}{\partial \xi} + \omega_{cd} q^{(1)} (v_{dx}^{(1)} \sin\theta - v_{dz}^{(1)} \cos\theta) = 0$$

$$\Lambda \frac{\partial}{\partial \xi}\left(-\frac{\Lambda^2 \sin\theta \sec^2\theta}{\omega_{cd}} \frac{\partial n_d^{(1)}}{\partial \xi}\right) + \omega_{cd}\left(q^{(1)} v_{dx}^{(1)} \sin\theta - q^{(1)} v_{dz}^{(1)} \cos\theta\right) = 0$$

$$-\frac{\Lambda^3 \sin\theta \sec^2\theta}{\omega_{cd}^2} \frac{\partial^2 n_d^{(1)}}{\partial \xi^2} + \left(v_{dx}^{(1)} \sin\theta - v_{dz}^{(1)} \cos\theta\right) = 0$$

$$-\frac{\sin\theta \sec^2\theta}{\omega_{cd}^2}\left[\cos^3\theta\left(\frac{\gamma_d \sigma_d}{\alpha_d} + \frac{1}{(1+\alpha_d \beta_{ch})}\right)^{3/2}\right] \frac{\partial^2 n_d^{(1)}}{\partial \xi^2} + \left(q^{(1)} v_{dx}^{(1)} \sin\theta - q^{(1)} v_{dz}^{(1)} \cos\theta\right) = 0$$

Taking derivative $'\frac{\partial}{\partial \zeta}'$ & Multiply $'\sin\theta'$,

$$-\frac{\sin^2\theta \cos\theta}{\omega_{cd}^2}\left(\frac{\gamma_d \sigma_d}{\alpha_d} + \frac{1}{(1+\alpha_d \beta_{ch})}\right)^{3/2} \frac{\partial^3 n_d^{(1)}}{\partial \xi^3} + \left[\left(q^{(1)} \frac{\partial v_{dx}^{(1)}}{\partial \zeta} + v_{dx}^{(1)} \frac{\partial q^{(1)}}{\partial \zeta}\right) \sin^2\theta - \left(q^{(1)} \frac{\partial v_{dz}^{(1)}}{\partial \zeta} + v_{dz}^{(1)} \frac{\partial q^{(1)}}{\partial \zeta}\right) \sin\right.$$

$$-\frac{\sin^2\theta \cos\theta}{\omega_{cd}^2}\left(\frac{\gamma_d \sigma_d}{\alpha_d} + \frac{1}{(1+\alpha_d \beta_{ch})}\right)^{3/2} \frac{\partial^3 n_d^{(1)}}{\partial \xi^3} + \left[\left\{\left(\frac{\alpha_d \beta_{ch}}{\alpha_d \beta_{ch}+1}\right) n_d^{(1)} \frac{\partial}{\partial \zeta}\left(-\Lambda n_d^{(1)}\right) + \left(-\Lambda n_d^{(1)}\right) \frac{\partial}{\partial \zeta}\left(\frac{\alpha_d \beta_{ch}}{\alpha_d \beta_{ch}+1}\right.\right.\right.$$

$$-\frac{\sin^2\theta \cos\theta}{\omega_{cd}^2}\left(\frac{\gamma_d \sigma_d}{\alpha_d} + \frac{1}{(1+\alpha_d \beta_{ch})}\right)^{3/2} \frac{\partial^3 n_d^{(1)}}{\partial \xi^3} + \left[\left\{\left(\frac{-\Lambda \alpha_d \beta_{ch}}{\alpha_d \beta_{ch}+1}\right) n_d^{(1)} \frac{\partial n_d^{(1)}}{\partial \zeta} + \left(\frac{-\Lambda \alpha_d \beta_{ch}}{\alpha_d \beta_{ch}+1}\right) n_d^{(1)} \frac{\partial n_d^{(1)}}{\partial \zeta}\right\} \sin^2\theta - \right.$$

$$-\frac{\sin^2\theta\cos\theta}{\omega_{cd}^2}\left(\frac{\gamma_d\sigma_d}{\alpha_d}+\frac{1}{(1+\alpha_d\beta_{ch})}\right)^{3/2}\frac{\partial^3 n_d^{(1)}}{\partial\xi^3}+\left[\left(\frac{-2\Lambda\alpha_d\beta_{ch}}{\alpha_d\beta_{ch}+1}\right)n_d^{(1)}\frac{\partial n_d^{(1)}}{\partial\zeta}\sin^2\theta+\left(\frac{2\Lambda\alpha_d\beta_{ch}\tan\theta}{\alpha_d\beta_{ch}+1}\right)n_d^{(1)}\frac{\partial n_d^{(1)}}{\partial\zeta}\right.$$

$$-\frac{\sin^2\theta\cos\theta}{\omega_{cd}^2}\left(\frac{\gamma_d\sigma_d}{\alpha_d}+\frac{1}{(1+\alpha_d\beta_{ch})}\right)^{3/2}\frac{\partial^3 n_d^{(1)}}{\partial\xi^3}+\left[\left(\frac{-2\Lambda\alpha_d\beta_{ch}}{\alpha_d\beta_{ch}+1}\right)n_d^{(1)}\frac{\partial n_d^{(1)}}{\partial\zeta}\sin^2\theta+\left(\frac{2\Lambda\alpha_d\beta_{ch}}{\alpha_d\beta_{ch}+1}\right)n_d^{(1)}\frac{\partial n_d^{(1)}}{\partial\zeta}\sin^2\right.$$

$$-\frac{\sin^2\theta\cos\theta}{\omega_{cd}^2}\left(\frac{\gamma_d\sigma_d}{\alpha_d}+\frac{1}{(1+\alpha_d\beta_{ch})}\right)^{3/2}\frac{\partial^3 n_d^{(1)}}{\partial\xi^3}=0 \qquad \text{(IX}'\text{)}$$

Now calculating (X),

$$\frac{\partial v_{dz}^{(1)}}{\partial\tau}+v_{dx}^{(1)}\frac{\partial v_{dz}^{(1)}}{\partial\xi}=-\omega_{cd}q^{(1)}v_{dy}^{(1)}\cos\theta$$

$$\frac{\partial}{\partial\tau}\left(-\Lambda n_d^{(1)}\tan\theta\right)+\left(-\Lambda n_d^{(1)}\right)\frac{\partial}{\partial\zeta}\left(-\Lambda n_d^{(1)}\tan\theta\right)+\omega_{cd}\left(\frac{\alpha_d\beta_{ch}}{\alpha_d\beta_{ch}+1}\right)n_d^{(1)}\left(-\frac{\Lambda^2\sin\theta\sec^2\theta}{\omega_{cd}}\frac{\partial n_d^{(1)}}{\partial\xi}\right)\cos\theta=0$$

$$-\Lambda\tan\theta\frac{\partial n_d^{(1)}}{\partial\tau}+\Lambda^2\tan\theta n_d^{(1)}\frac{\partial n_d^{(1)}}{\partial\xi}-\frac{\Lambda^2\alpha_d\beta_{ch}\sin\theta\sec^2\theta\cos\theta}{\alpha_d\beta_{ch}+1}n_d^{(1)}\frac{\partial n_d^{(1)}}{\partial\xi}=0$$

$$-\Lambda\tan\theta\frac{\partial n_d^{(1)}}{\partial\tau}+\left[\Lambda^2\tan\theta-\frac{\Lambda^2\alpha_d\beta_{ch}\sin\theta\sec\theta}{\alpha_d\beta_{ch}+1}\right]n_d^{(1)}\frac{\partial n_d^{(1)}}{\partial\xi}=0$$

Dividing by $'-\Lambda\tan\theta'$,

$$\frac{\partial n_d^{(1)}}{\partial\tau}+\left[-\Lambda+\frac{\Lambda\alpha_d\beta_{ch}}{\alpha_d\beta_{ch}+1}\right]n_d^{(1)}\frac{\partial n_d^{(1)}}{\partial\xi}=0 \qquad \text{(X}'\text{)}$$

Calculating (XI),

$$\frac{1}{2\gamma}\left(\delta_++\frac{\delta_-}{\sigma_-^2}-\frac{1}{\sigma_+^2}\right)\varphi^{(1)^2}-\alpha_d q^{(1)}n_d^{(1)}=0$$

$$\frac{1}{2\gamma}\left(\delta_+ + \frac{\delta_-}{\sigma_-^2} - \frac{1}{\sigma_+^2}\right)\frac{\alpha_d^2}{(\alpha_d\beta_{ch}+1)^2} n_d^{2(1)} - \alpha_d\left(\frac{\alpha_d\beta_{ch}}{\alpha_d\beta_{ch}+1}\right) n_d^{(1)} n_d^{(1)} = 0$$

$$\frac{\alpha_d^2\left(\delta_+ + \frac{\delta_-}{\sigma_-^2} - \frac{1}{\sigma_+^2}\right) n_d^{2(1)}}{2\gamma\left(\alpha_d\beta_{ch}+1\right)^2} - \frac{\alpha_d^2\beta_{ch} n_d^{2(1)}}{\alpha_d\beta_{ch}+1} = 0$$

Taking $'\frac{\partial}{\partial\zeta}'$,

$$\frac{2\alpha_d^2\left(\delta_+ + \frac{\delta_-}{\sigma_-^2} - \frac{1}{\sigma_+^2}\right) n_d^{(1)}}{2\gamma\left(\alpha_d\beta_{ch}+1\right)^2}\frac{\partial n_d^{(1)}}{\partial\xi} - \frac{2\alpha_d^2\beta_{ch} n_d^{(1)}}{\alpha_d\beta_{ch}+1}\frac{\partial n_d^{(1)}}{\partial\xi} = 0$$

$$\left[\frac{\alpha_d^2\left(\delta_+ + \frac{\delta_-}{\sigma_-^2} - \frac{1}{\sigma_+^2}\right)}{\gamma\left(\alpha_d\beta_{ch}+1\right)^2} - \frac{2\alpha_d^2\beta_{ch}}{\alpha_d\beta_{ch}+1}\right] n_d^{(1)}\frac{\partial n_d^{(1)}}{\partial\xi} = 0$$

$$\alpha_d^2\left[\frac{2\beta_{ch}}{\alpha_d\beta_{ch}+1} - \frac{\left(\delta_+ + \frac{\delta_-}{\sigma_-^2} - \frac{1}{\sigma_+^2}\right)}{\gamma\left(\alpha_d\beta_{ch}+1\right)^2}\right] n_d^{(1)}\frac{\partial n_d^{(1)}}{\partial\xi} = 0$$

$$\left[\frac{2\beta_{ch}}{\alpha_d\beta_{ch}+1} - \frac{\left(\delta_+ + \frac{\delta_-}{\sigma_-^2} - \frac{1}{\sigma_+^2}\right)}{\gamma\left(\alpha_d\beta_{ch}+1\right)^2}\right] n_d^{(1)}\frac{\partial n_d^{(1)}}{\partial\xi} = 0 \qquad \text{(XI}'\text{)}$$

Now calculating (XII),

$$\Lambda\omega_{ch}\frac{\partial q^{(1)}}{\partial\xi} + \frac{(\sigma_+\beta_{ch})z^2(1+B_-)q^{(1)^2}}{2(1+\sigma_+ + \gamma_{\overline{2}})} - \frac{(1-\sigma_+^2(1+B_-))}{\sigma_+^2}\varphi^{(1)^2} - \frac{z(1-\sigma_+(z+\sigma_+)(1+B_-))}{\sigma_+(z+\sigma_+)}q^{(1)}\varphi^{(1)} = 0$$

$$\Lambda\omega_{ch}\frac{\partial}{\partial\xi}\left(\frac{\alpha_d\beta_{ch}}{\alpha_d\beta_{ch}+1}\right)n_d^{(1)} + \frac{\sigma_+\beta_{ch}}{2(1+\sigma_+ + \gamma_{\overline{2}})}\left[z^2(1+B_-)\left(\frac{\alpha_d\beta_{ch}}{\alpha_d\beta_{ch}+1}\right)^2 n_d^{2(1)} - \frac{(1-\sigma_+^2(1+B_-))}{\sigma_+^2}\left(\frac{\alpha_d^2}{(\alpha_d\beta_{ch}+1\right.\right.$$

$$\frac{\Lambda\omega_{ch}\alpha_d\beta_{ch}}{(\alpha_d\beta_{ch}+1)}\frac{\partial n_d^{(1)}}{\partial\xi} + \frac{\sigma_+\beta_{ch}\alpha_d^2}{2(1+\sigma_+ + \gamma_{\overline{2}})\left(\alpha_d\beta_{ch}+1\right)^2}\left[z^2\beta_{ch}^2\left(1+B_-\right) - \frac{(1-\sigma_+^2(1+B_-))}{\sigma_+^2} + \frac{2z(1-\sigma_+(z+\sigma_+)(1\right.}{\sigma_+(z+\sigma_+)}$$

Dividing by $'\Lambda'$ & Taking derivative $'\frac{\partial}{\partial\zeta}'$,

$$\frac{\omega_{ch}\alpha_d\beta_{ch}}{(\alpha_d\beta_{ch}+1)}\frac{\partial^2 n_d^{(1)}}{\partial\xi^2}+\frac{2\sigma_+\beta_{ch}\alpha_d^2}{2\Lambda(1+\sigma_++\gamma_{\overline{2}})(\alpha_d\beta_{ch}+1)^2}\left[z^2\beta_{ch}^2(1+B_-)-\frac{(1-\sigma_+^2(1+B_-))}{\sigma_+^2}+\frac{2z\beta_{ch}(1-\sigma_+(z+\sigma}{\sigma_+(z+\sigma}\right.$$

Dividing by $(\alpha_d\beta_{ch}+1)$,

$$\frac{\omega_{ch}\alpha_d\beta_{ch}}{(\alpha_d\beta_{ch}+1)^2}\frac{\partial^2 n_d^{(1)}}{\partial\xi^2}+\frac{2\sigma_+\beta_{ch}\alpha_d^2}{2\Lambda(1+\sigma_++\gamma_{\overline{2}})(\alpha_d\beta_{ch}+1)^3}\left[z^2\beta_{ch}^2(1+B_-)-\frac{(1-\sigma_+^2(1+B_-))}{\sigma_+^2}+\frac{2z\beta_{ch}(1-\sigma_+(z+}{\sigma_+(z+}\right. \tag{XII'}$$

4.3.1 Set Of Equaions:

$$\frac{\partial n_d^{(1)}}{\partial\tau}-2\Lambda n_d^{(1)}\frac{\partial}{\partial\xi}n_d^{(1)}=0 \tag{VII$prime$}$$

$$\frac{\partial n_d^{(1)}}{\partial\tau}+\left[-\Lambda+\frac{\alpha_d\beta_{ch}}{\Lambda(\alpha_d\beta_{ch}+1)^2}-\frac{\gamma_d(\gamma_d-2)\sigma_d}{\Lambda\alpha_d}-\frac{\Lambda\sin^2\theta\sec^2\theta\alpha_d\beta_{ch}}{(\alpha_d\beta_{ch}+1)}\right]n_d^{(1)}\frac{\partial n_d^{(1)}}{\partial\xi}=0 \tag{VIII'}$$

$$-\frac{\sin^2\theta\cos\theta}{\omega_{cd}^2}\left(\frac{\gamma_d\sigma_d}{\alpha_d}+\frac{1}{(1+\alpha_d\beta_{ch})}\right)^{3/2}\frac{\partial^3 n_d^{(1)}}{\partial\xi^3}=0 \tag{IX'}$$

$$\frac{\partial n_d^{(1)}}{\partial\tau}+\left[-\Lambda+\frac{\Lambda\alpha_d\beta_{ch}}{\alpha_d\beta_{ch}+1}\right]n_d^{(1)}\frac{\partial n_d^{(1)}}{\partial\xi}=0 \tag{X'}$$

$$\left[\frac{2\alpha_d\beta_{ch}}{\alpha_d\beta_{ch}+1}-\frac{\alpha_d\left(\delta_++\frac{\delta_-}{\sigma_-^2}-\frac{1}{\sigma_+^2}\right)}{\gamma(\alpha_d\beta_{ch}+1)^2}\right]n_d^{(1)}\frac{\partial n_d^{(1)}}{\partial\xi}=0 \tag{XI'}$$

$$\frac{\omega_{ch}\alpha_d\beta_{ch}}{(\alpha_d\beta_{ch}+1)^2}\frac{\partial^2 n_d^{(1)}}{\partial\xi^2}+\frac{2\sigma_+\beta_{ch}\alpha_d^2}{2\Lambda(1+\sigma_++\gamma_{\overline{2}})(\alpha_d\beta_{ch}+1)^3}\left[z^2\beta_{ch}^2(1+B_-)-\frac{(1-\sigma_+^2(1+B_-))}{\sigma_+^2}+\frac{2z\beta_{ch}(1-\sigma_+(z+}{\sigma_+(z+}\right. \tag{XII'}$$

Subtract (X′) from (VIII′)

$$\frac{\partial n_d^{(1)}}{\partial \tau}+\left[-\Lambda+\frac{\alpha_d\beta_{ch}}{\Lambda\left(\alpha_d\beta_{ch}+1\right)^2}-\frac{\gamma_d(\gamma_d-2)\sigma_d}{\Lambda\alpha_d}-\frac{\Lambda\sin^2\theta\sec^2\theta\alpha_d\beta_{ch}}{(\alpha_d\beta_{ch}+1)}\right]n_d^{(1)}\frac{\partial n_d^{(1)}}{\partial \xi}-\left[\frac{\partial n_d^{(1)}}{\partial \tau}+\left[-\Lambda+\frac{\Lambda\alpha_d\beta_{ch}}{\alpha_d\beta_{ch}+1}\right]\right.$$

$$\frac{\partial n_d^{(1)}}{\partial \tau}-\frac{\partial n_d^{(1)}}{\partial \tau}+\left[-\Lambda+\frac{\alpha_d\beta_{ch}}{\Lambda\left(\alpha_d\beta_{ch}+1\right)^2}-\frac{\gamma_d(\gamma_d-2)\sigma_d}{\Lambda\alpha_d}-\frac{\Lambda\sin^2\theta\sec^2\theta\alpha_d\beta_{ch}}{(\alpha_d\beta_{ch}+1)}+\Lambda-\frac{\Lambda\alpha_d\beta_{ch}}{\alpha_d\beta_{ch}+1}\right]n_d^{(1)}\frac{\partial n_d^{(1)}}{\partial \xi}=0$$

$$\left[\frac{\alpha_d\beta_{ch}}{\Lambda\left(\alpha_d\beta_{ch}+1\right)^2}-\frac{\Lambda\alpha_d\beta_{ch}}{\alpha_d\beta_{ch}+1}-\frac{\gamma_d(\gamma_d-2)\sigma_d}{\Lambda\alpha_d}-\frac{\Lambda\sin^2\theta\sec^2\theta\alpha_d\beta_{ch}}{(\alpha_d\beta_{ch}+1)}\right]n_d^{(1)}\frac{\partial n_d^{(1)}}{\partial \xi}=0$$

Adding (XI$'$) in above equaiton,

$$\left[\frac{\alpha_d\beta_{ch}}{\Lambda\left(\alpha_d\beta_{ch}+1\right)^2}-\frac{\Lambda\alpha_d\beta_{ch}}{\alpha_d\beta_{ch}+1}-\frac{\gamma_d(\gamma_d-2)\sigma_d}{\Lambda\alpha_d}-\frac{\Lambda\sin^2\theta\sec^2\theta\alpha_d\beta_{ch}}{(\alpha_d\beta_{ch}+1)}\right]n_d^{(1)}\frac{\partial n_d^{(1)}}{\partial \xi}+\left[\frac{2\beta_{ch}}{\Lambda\left(\alpha_d\beta_{ch}+1\right)}-\frac{\left(\delta_++\frac{\delta_-}{\sigma_-^2}\right.}{\gamma\Lambda\left(\alpha_d\beta_{ch}\right.}\right.$$

$$\left[\frac{\alpha_d\beta_{ch}}{\Lambda\left(\alpha_d\beta_{ch}+1\right)^2}-\frac{\Lambda\alpha_d\beta_{ch}}{\alpha_d\beta_{ch}+1}-\frac{\gamma_d(\gamma_d-2)\sigma_d}{\Lambda\alpha_d}-\frac{\Lambda\sin^2\theta\sec^2\theta\alpha_d\beta_{ch}}{(\alpha_d\beta_{ch}+1)}+\frac{2\beta_{ch}}{\Lambda\left(\alpha_d\beta_{ch}+1\right)}-\frac{\left(\delta_++\frac{\delta_-}{\sigma_-^2}-\frac{1}{\sigma_+^2}\right)}{\Lambda\gamma\left(\alpha_d\beta_{ch}+1\right)^2}\right]n_d^{(1}$$

Adding (XII$'$),

$$\frac{\omega_{ch}\alpha_d\beta_{ch}}{\left(\alpha_d\beta_{ch}+1\right)^2}\frac{\partial^2 n_d^{(1)}}{\partial \xi^2}+\left[\frac{\alpha_d\beta_{ch}}{\Lambda\left(\alpha_d\beta_{ch}+1\right)^2}-\frac{\Lambda\alpha_d\beta_{ch}}{\alpha_d\beta_{ch}+1}-\frac{\gamma_d(\gamma_d-2)\sigma_d}{\Lambda\alpha_d}-\frac{\Lambda\sin^2\theta\sec^2\theta\alpha_d\beta_{ch}}{(\alpha_d\beta_{ch}+1)}+\frac{2\beta_{ch}}{\Lambda\left(\alpha_d\beta_{ch}+1\right)}-\right.$$

Multiply $'\cos^2\theta'$,

$$\frac{\omega_{ch}\alpha_d\beta_{ch}\cos^2\theta}{\left(\alpha_d\beta_{ch}+1\right)^2}\frac{\partial^2 n_d^{(1)}}{\partial \xi^2}-\left[\frac{\Lambda\alpha_d\beta_{ch}\cos^2\theta}{\alpha_d\beta_{ch}+1}-\frac{\alpha_d\beta_{ch}\cos^2\theta}{\Lambda\left(\alpha_d\beta_{ch}+1\right)^2}+\frac{\Lambda\sin^2\theta\alpha_d\beta_{ch}}{(\alpha_d\beta_{ch}+1)}-\frac{\gamma_d(\gamma_d-2)\sigma_d\cos^2\theta}{\Lambda\alpha_d}-\frac{2\beta_{ch}\cos^2\theta}{\alpha_d\beta_{ch}+1}\right.$$

Adding (IX)′ & Divide by ′2′,

$$\frac{\omega_{ch}\alpha_d\beta_{ch}\cos^2\theta}{2\left(\alpha_d\beta_{ch}+1\right)^2}\frac{\partial^2 n_d^{(1)}}{\partial\xi^2}-\frac{\sin^2\theta\cos\theta}{\omega_{cd}^2}\left(\frac{\gamma_d\sigma_d}{\alpha_d}+\frac{1}{(1+\alpha_d\beta_{ch})}\right)^{3/2}\frac{\partial^3 n_d^{(1)}}{\partial\xi^3}-\left[\frac{\Lambda\alpha_d\beta_{ch}\cos^2\theta}{(\alpha_d\beta_{ch}+1)}-\frac{\alpha_d\beta_{ch}\cos^2\theta}{2\Lambda\left(\alpha_d\beta_{ch}+1\right)^2}+\frac{\Lambda}{2(}\right.$$

Adding (VII)′ to above equation,

$$\frac{\partial n_d^{(1)}}{\partial\tau}-\frac{\omega_{ch}\alpha_d\beta_{ch}\cos^2\theta}{2\left(\alpha_d\beta_{ch}+1\right)^2}\frac{\partial^2 n_d^{(1)}}{\partial\xi^2}-\frac{\sin^2\theta\cos\theta}{\omega_{cd}^2}\left(\frac{\gamma_d\sigma_d}{\alpha_d}+\frac{1}{(1+\alpha_d\beta_{ch})}\right)^{3/2}\frac{\partial^3 n_d^{(1)}}{\partial\xi^3}-\left[\begin{array}{c} 2\Lambda+\frac{\Lambda\alpha_d\beta_{ch}\cos}{(\alpha_d\beta_{ch}+} \\ +\frac{\alpha_d\left(\delta_++\frac{\delta_-}{\sigma_-^2}-\frac{1}{\sigma_+^2}\right)\cos^2\theta}{2\gamma\Lambda(\alpha_d\beta_{ch}+1)^2}+\frac{}{2\Lambda(1-} \end{array}\right.$$

Now solving,

$$\frac{\sigma_+\beta_{ch}\alpha_d^2\cos^2\theta}{2\Lambda(1+\sigma_++\gamma_{\overline{2}})\left(\alpha_d\beta_{ch}+1\right)^3}\left\{z^2\beta_{ch}^2\left(1+B_-\right)-\frac{(1-\sigma_+^2(1+B_-))}{\sigma_+^2}+\frac{2z\beta_{ch}(1-\sigma_+(z+\sigma_+)(1+B_-))}{\sigma_+(z+\sigma_+)}\right\}$$

$$\frac{z^2\sigma_+\beta_{ch}^3\alpha_d^2\cos^2\theta\left(1+B_-\right)}{2\Lambda(1+\sigma_++\gamma_{\overline{2}})\left(\alpha_d\beta_{ch}+1\right)^3}+\frac{\sigma_+\beta_{ch}\alpha_d^2\cos^2\theta}{2\Lambda(1+\sigma_++\gamma_{\overline{2}})\left(\alpha_d\beta_{ch}+1\right)^3}\left\{-\frac{1}{\sigma_+^2}+\left(1+B_-\right)+\frac{2z\beta_{ch}}{\sigma_+(z+\sigma_+)}-2z\beta_{ch}(1+B\right.$$

$$\frac{z^2\sigma_+\beta_{ch}^3\alpha_d^2\cos^2\theta\left(1+B_-\right)}{2\Lambda(1+\sigma_++\gamma_{\overline{2}})\left(\alpha_d\beta_{ch}+1\right)^3}-\frac{\beta_{ch}\alpha_d^2\cos^2\theta}{2\Lambda\sigma_+(1+\sigma_++\gamma_{\overline{2}})\left(\alpha_d\beta_{ch}+1\right)^3}+\frac{\sigma_+\beta_{ch}\alpha_d^2\cos^2\theta(1+B_-)}{2\Lambda(1+\sigma_++\gamma_{\overline{2}})\left(\alpha_d\beta_{ch}+1\right)^3}+\frac{}{2\Lambda\sigma_+(1+\sigma}$$

$$\frac{z^2\sigma_+\beta_{ch}^3\alpha_d^2\cos^2\theta\left(1+B_-\right)}{2\Lambda(1+\sigma_++\gamma_{\overline{2}})\left(\alpha_d\beta_{ch}+1\right)^3}+\frac{\sigma_+\beta_{ch}\alpha_d^2\cos^2\theta(1+B_-)}{2\Lambda(1+\sigma_++\gamma_{\overline{2}})\left(\alpha_d\beta_{ch}+1\right)^3}-\frac{2z\sigma_+\beta_{ch}^2\alpha_d^2\cos^2\theta(1+B_-)}{2\Lambda(1+\sigma_++\gamma_{\overline{2}})\left(\alpha_d\beta_{ch}+1\right)^3}+\frac{2z}{2\Lambda\sigma_+(1+\sigma_+}$$

So,

$$\frac{\sigma_+\beta_{ch}\alpha_d^2\cos^2\theta}{2\Lambda(1+\sigma_++\gamma_{\overline{2}})\left(\alpha_d\beta_{ch}+1\right)^3}\left\{\left(1+B_-\right)\left(z^2\beta_{ch}^2+1-2z\beta_{ch}\right)\right\}+\frac{2z\sigma_+\beta_{ch}\alpha_d^2\cos^2\theta}{2\Lambda\sigma_+(1+\sigma_++\gamma_{\overline{2}})\left(\alpha_d\beta_{ch}+1\right)^3\left(z+\sigma_+\right)}\Bigg(\frac{-}{\sigma}$$

$$\frac{\sigma_+\beta_{ch}\alpha_d^2\cos^2\theta}{2\Lambda(1+\sigma_++\gamma_{\overline{2}})\left(\alpha_d\beta_{ch}+1\right)^3}\left\{\left(1+B_-\right)\left(z\beta_{ch}-1\right)^2+\frac{2z\beta_{ch}}{\sigma_+(z+\sigma_+)}-\frac{1}{\sigma_+^2}\right\}$$

$$\frac{\sigma_+\beta_{ch}\alpha_d^2\cos^2\theta}{2\Lambda(1+\sigma_++\gamma_{\overline{2}})\left(\alpha_d\beta_{ch}+1\right)^3}\left(C\right)=A$$

where,

$$C=\left(1+B_-\right)\left(z\beta_{ch}-1\right)^2+\frac{2z\beta_{ch}}{\sigma_+(z+\sigma_+)}-\frac{1}{\sigma_+^2}$$

Now,

$$2\Lambda=\frac{2\Lambda^2}{\Lambda}=\frac{4\Lambda^2}{2\Lambda}=\frac{4}{2\Lambda}\cos^2\theta\left(\frac{\gamma_d\sigma_d}{\alpha_d}+\frac{1}{(1+\alpha_d\beta_{ch})}\right)=\frac{4\cos^2\theta}{2\Lambda}\frac{\gamma_d\sigma_d}{\alpha_d}+\frac{4\cos^2\theta}{2\Lambda\left(1+\alpha_d\beta_{ch}\right)}$$

$$2\Lambda=\frac{4\cos^2\theta}{2\Lambda}\frac{\gamma_d\sigma_d}{\alpha_d}+\frac{4\cos^2\theta}{2\Lambda\left(1+\alpha_d\beta_{ch}\right)}\tag{(i)'}$$

Also,

$$\frac{\Lambda\alpha_d\beta_{ch}\cos^2\theta}{2\left(\alpha_d\beta_{ch}+1\right)^2}+\frac{\Lambda\sin^2\theta\alpha_d\beta_{ch}}{2\left(\alpha_d\beta_{ch}+1\right)}=\frac{\Lambda\alpha_d\beta_{ch}}{2\left(\alpha_d\beta_{ch}+1\right)^2}\left(\cos^2\theta+\sin^2\theta\right)$$

$$=\frac{\Lambda\alpha_d\beta_{ch}}{2\left(\alpha_d\beta_{ch}+1\right)^2}=\frac{\Lambda^2\alpha_d\beta_{ch}}{2\Lambda\left(\alpha_d\beta_{ch}+1\right)^2}=\frac{\alpha_d\beta_{ch}}{2\Lambda\left(\alpha_d\beta_{ch}+1\right)^2}\cos^2\theta\left(\frac{\gamma_d\sigma_d}{\alpha_d}+\frac{1}{(1+\alpha_d\beta_{ch})}\right)$$

$$\frac{\cos^2\theta\gamma_d\sigma_d\beta_{ch}}{2\Lambda\left(\alpha_d\beta_{ch}+1\right)}+\frac{\cos^2\theta\alpha_d\beta_{ch}}{2\Lambda\left(\alpha_d\beta_{ch}+1\right)^2}=\frac{\Lambda^2\alpha_d\beta_{ch}}{2\Lambda\left(\alpha_d\beta_{ch}+1\right)^2}\left(\cos^2\theta+\sin^2\theta\right)\tag{(ii)'}$$

Putting (i)$'$ & (ii)$'$ in above equation,

$$A = \frac{4\cos^2\theta}{2\Lambda}\frac{\gamma_d\sigma_d}{\alpha_d} + \frac{4\cos^2\theta}{2\Lambda\left(1+\alpha_d\beta_{ch}\right)} + \frac{\cos^2\theta\gamma_d\sigma_d\beta_{ch}}{2\Lambda\left(\alpha_d\beta_{ch}+1\right)} - \frac{2\cos^2\theta\alpha_d\beta_{ch}}{2\Lambda\left(1+\alpha_d\beta_{ch}\right)^2} + \frac{\cos^2\theta\gamma_d^2\sigma_d}{2\Lambda\alpha_d} - \frac{2\cos^2\theta\gamma_d\sigma_d}{2\Lambda\alpha_d}$$

$$= \frac{4\cos^2\theta}{2\Lambda\left(1+\alpha_d\beta_{ch}\right)} + \frac{\cos^2\theta\gamma_d\sigma_d\beta_{ch}}{2\Lambda\left(\alpha_d\beta_{ch}+1\right)} - \frac{2\cos^2\theta\alpha_d\beta_{ch}}{2\Lambda\left(1+\alpha_d\beta_{ch}\right)^2} + \frac{\cos^2\theta\gamma_d\sigma_d\left(\gamma_d+2\right)}{2\Lambda\alpha_d}$$

Putting value in equation-( )

$$\frac{\partial n_d^{(1)}}{\partial\tau} - \mu_{ch}\frac{\partial^2 n_d^{(1)}}{\partial\xi^2} - \beta\frac{\partial^3 n_d^{(1)}}{\partial\xi^3} - \Bigg[\frac{4\cos^2\theta}{2\Lambda\left(1+\alpha_d\beta_{ch}\right)} + \frac{\cos^2\theta\gamma_d\sigma_d\beta_{ch}}{2\Lambda\left(\alpha_d\beta_{ch}+1\right)} - \frac{2\cos^2\theta\alpha_d\beta_{ch}}{2\Lambda\left(1+\alpha_d\beta_{ch}\right)^2} + \frac{\cos^2\theta\gamma_d\sigma_d\left(\gamma_d+2\right)}{2\Lambda\alpha_d} + \frac{\alpha_d}{}$$

$$\frac{\partial n_d^{(1)}}{\partial\tau} - \mu_{ch}\frac{\partial^2 n_d^{(1)}}{\partial\xi^2} - \beta\frac{\partial^3 n_d^{(1)}}{\partial\xi^3} - \Bigg[\frac{\cos^2\theta}{2\Lambda}\Bigg(\frac{\gamma_d\sigma_d\left(\gamma_d+2\right)}{\alpha_d} + \frac{\gamma_d\sigma_d\beta_{ch}}{\left(\alpha_d\beta_{ch}+1\right)} - \frac{2\alpha_d\beta_{ch}}{\left(1+\alpha_d\beta_{ch}\right)^2} + \frac{4}{\left(\alpha_d\beta_{ch}+1\right)} + \frac{\alpha_d\left(\delta_+ +\right.}{\gamma\left(\alpha_d\beta\right.}$$

$$\frac{\partial n_d^{(1)}}{\partial\tau} - \mu_{ch}\frac{\partial^2 n_d^{(1)}}{\partial\xi^2} - \beta\frac{\partial^3 n_d^{(1)}}{\partial\xi^3} - \frac{\cos^2\theta}{2\Lambda}\Bigg[\frac{\gamma_d\sigma_d\left(\gamma_d+2\right)}{\alpha_d} + \frac{\gamma_d\sigma_d\beta_{ch}}{\left(\alpha_d\beta_{ch}+1\right)} + \frac{1}{\left(1+\alpha_d\beta_{ch}\right)^2}\Bigg\{-2\alpha_d\beta_{ch} + \frac{\alpha_d\left(\delta_+ + \frac{\delta_-}{\sigma_-^2} -\right.}{\gamma}$$

$$\frac{\partial n_d^{(1)}}{\partial\tau} - \mu_{ch}\frac{\partial^2 n_d^{(1)}}{\partial\xi^2} - \beta\frac{\partial^3 n_d^{(1)}}{\partial\xi^3} - \alpha n_d^{(1)}\frac{\partial n_d^{(1)}}{\partial\xi} = 0$$

$$\frac{\partial n_d^{(1)}}{\partial\tau} - \alpha n_d^{(1)}\frac{\partial n_d^{(1)}}{\partial\xi} - \beta\frac{\partial^3 n_d^{(1)}}{\partial\xi^3} = \mu_{ch}\frac{\partial^2 n_d^{(1)}}{\partial\xi^2}. \tag{4.24}$$

The coefficients of nonlinearity α, dispersion β and the Burger term μ_{ch} are as follows:

$$\alpha = \frac{\cos\theta}{\left(1+\alpha_d\beta_{ch}\right)^{1/2}}\left[\begin{array}{c}\frac{\gamma_d(\gamma_d+1)\sigma_d}{\alpha_d} + \frac{1}{(1+\alpha_d\beta_{ch})^2} \\ \left(\frac{3\gamma+\delta_+ + \frac{\delta_-}{\sigma_-^2} - \frac{1}{\sigma_+^2}}{\gamma} + \frac{\sigma_+\beta_{ch}\alpha_d C}{(1+\sigma_+\beta_{ch})(1+\sigma_+ + \gamma_{\overline{2}})}\right)\end{array}\right], \tag{4.25}$$

where

$$C = (1 + B_-)(z\beta_{ch} - 1)^2 + \frac{2z\beta_{ch}}{\sigma_+(z + \sigma_+)} - \frac{1}{\sigma_+^2}, \tag{4.26}$$

$$\beta = \frac{\sin^2\theta\cos\theta}{2\omega_{cd}^2}\left(\frac{\gamma_d\sigma_d}{\alpha_d} + \frac{1}{1 + \beta_{ch}\alpha_d}\right)^{3/2}, \tag{4.27}$$

and

$$\mu_{ch} = \frac{\alpha_d\beta_{ch}\omega_{ch}\cos^2\theta}{2(1 + \beta_{ch}\alpha_d)^2}. \tag{4.28}$$

This demonstrates that the obliqueness of the magnetic field is proportional to the coefficient of the nonlinear factor. The expression indicates that the dispersive term is inversely proportional to the magnetic field's magnitude and proportionate to the magnetic field's obliqueness. Therefore, only the external magnetic field's presence causes the dissipative term to appear. For parallel propagation (■=0), the dispersive term disappears when ■=0, and the well-known Burger equation governs the nonlinear wave:

$$\frac{\partial n_d^{(1)}}{\partial\tau} - \alpha n_d^{(1)}\frac{\partial n_d^{(1)}}{\partial\xi} = \mu_{ch}\frac{\partial^2 n_d^{(1)}}{\partial\xi^2}. \tag{4.29}$$

The expression for μ_{ch} [Eq. (6.26)], shows that the Burger term is proportional to $\omega_{ch} = (\omega_{pd}/\nu_{ch})$, and arises due to non-steady dust charge variations. Also, note that $\Delta = 0 \Longrightarrow \mu_{ch} = 0$. This is expected as $\Delta = 0$ implies the absence of charged dust grains and consequently no charge fluctuation. Hence, the Burger term, which is responsible for the generation of shock waves, originates from the non-steady dust charge variation under the assumption that ω_{ch} is finite. Also note that the coefficient of the Burger term is proportional to $\cos^2\theta$ (the other plasma parameter remains constant). Thus, the dissipative term depends only on the obliqueness of the magnetic field.

It is interesting to note that for *adiabatic* dust charge variation [33] $\omega_{ch} \approx 0$ [e.g., for typical laboratory dusty plasma, the dust oscillation frequency $\omega_{pd} \approx 10^2$ s^{-1} and the dust charging frequency $\nu_{ch} \approx 10^8$ s^{-1} [34], implies $\omega_{ch}(\approx 10^{-6}) \approx 0$] and the charging equation (6.9) is approximated by $I_e + I_+ + I_- = 0$, so that the charge on the dust grains instantaneously reaches its equilibrium value and, hence, does not play any dissipative role. Thus, in this case, $\mu_{ch} = 0$, i.e. the Burger term vanishe, and thereby the nonlinear dynamics of the DAW is

governed by the well-known Korteweg-de Vries equation that exhibits only the soliton solution instead of the shock wave.

4.4 Steady state solution: generation of shock waves

It is well known that the KdV Burger equation describes a shock wave profile. The criterium for the formation of a shock wave is that the coefficient of the Burger term, which aries due to nonsteady dust charge variations, should be positive (here $\mu_{ch} > 0$). Otherwise, it would not be possible to get a stable solution of the Burger equation. A particular solution of the above KdV Burger equation [Eq. (6.22)], is of the following form (Jeffrey and Xu 1989):

$$n_d^{(1)}(\xi,\tau) = -\frac{3\mu_{ch}^2}{25\alpha\beta}\left[1+\tanh\frac{\mu_{ch}}{10\beta}\left(\frac{6\mu_{ch}^2}{25\beta}\tau-\xi\right)\right]^2. \tag{4.30}$$

On the other hand, for parallel propagation ($\theta = 0$), one can easily find the following analytic solution of the Burger equation:

$$n_d^{(1)}(\xi,\tau) = N\left[1+\tanh\left(\frac{\eta}{L_w}\right)\right]^2, \tag{4.31}$$

Chapter 5

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